Probability, midterm exam practice.

The midterm will consist in eight such exercises.

Exercise 1. Prove that

$$\mathbb{P}(E\cup F\cup G) = \mathbb{P}(E)+\mathbb{P}(F)+\mathbb{P}(G)-\mathbb{P}(E^cFG)-\mathbb{P}(EF^cG)-\mathbb{P}(EFG^c)-2\mathbb{P}(EFG)$$

Exercise 2. Roll three dice. Find the probability that there are at least two five given that there is at least one five.

Exercise 3. Find the conditional probability that a uniform poker hand has at least 3 aces given that it has at least 2.

Exercise 4. In New York, 51% of the adults are females.One adult is randomly selected for a survey involving credit card usage. It is later learned that the selected survey subject was smoking. Also, 9.5% of males smoke, whereas 1.7% of females smoke. Use this additional information to find the probability that the selected subject is a male.

Exercise 5. Assume the sample space consists of a countably infinite number of points. Show that not all points can be equally likely.

Exercise 6. Thirty people are invited to a party. Each person accepts the invitation, independently of all others, with probability 1/4. Let X be the number of accepted invitations. Compute the following $\mathbb{E}(X)$, $\mathrm{Var}(X)$, $\mathbb{E}(X^2+X+5)$.

Exercise 7. Suppose that the sample space S contains three elements $\{1,2,3\}$, with probabilities 0.5, 0.2, and 0.3 respectively. Suppose $X(s)=s^2-4$ for $s\in S$. Compute $\mathbb{E}(|X|)$ and $\mathrm{Var}(|X|)$.

Exercise 8. Suppose X is Poissonian random variable with parameter $\lambda_1=1$, Y is an independent Poissonian random variable with $\lambda_2=3$, and Z is a Poissonian random variable with parameter $\lambda_3=4$. Assume X and Y and Z are independent. Compute $\mathbb{P}(X+Y+Z=8)$, $\mathbb{E}(X^2Y^2Z)$.

Exercise 9. I have noticed that during every given minute, there is a 1/1000 chance that my instagram page will get a *like*, independently of what happens during any other minute. Let L be the total number of likes during 24 hours. Compute exactly the expectation and variance of L, and the probability that $L=5$. Compute approximatively he probability that L is at least 2.

Exercise 10. Consider an infinite sequence of independent tosses of a coin that comes up heads with probability p. What is the probability that the fifth head occurs on the twentieth toss.

Exercise 11. Thirty people have independent birthdays uniformly among 365 possible days. Let X be the number of pairs of people with the same birthday (if everyone has the same birthday, then $X = 30 \times 29/2$). Let Y be the number ways of choosing a triple of three people that share a birthday. Compute $\mathbb{E}(X)$, $\mathrm{Var}(X)$, $\mathbb{E}(Y)$.