Probability, homework 4, due October 4th.

From *A first course in probability*, ninth edition, by Sheldon Ross.

Exercise 1. There are 15 tennis balls in a box, of which 9 have not previously been used. Three of the balls are randomly chosen, played with, and then returned to the box. Later, another 3 balls are randomly chosen from the box. Find the probability that none of these balls has ever been used.

Exercise 2. Suppose that you continually collect coupons and that there are m different types. Suppose also that each time a new coupon is obtained, it is a type i coupon with probability p_i, $i = 1, \ldots, m$. Suppose that you have just collected your nth coupon. What is the probability that it is a new type?

Exercise 3. There is a 50-50 chance that the queen carries the gene for hemophilia. If she is a carrier, then each prince has a 50-50 chance of having hemophilia. If the queen has had three princes without the disease, what is the probability that the queen is a carrier? If there is a fourth prince, what is the probability that he will have hemophilia?

Exercise 4. A and B alternate rolling a pair of dice, stopping either when A rolls the sum 9 or when B rolls the sum 6. Assuming that A rolls first, find the probability that the final roll is made by A.

Exercise 5. A and B play a series of games. Each game is independently won by A with probability p and by B with probability $1 - p$. They stop when the total number of wins of one of the players is two greater than that of the other player. The player with the greater number of total wins is declared the winner of the series.

(a) Find the probability that a total of 4 games are played.

(b) Find the probability that A is the winner of the series.

Exercise 6. Let $S = \{1, 2, \ldots, n\}$ and suppose that A and B are, independently, equally likely to be any of the 2^n subsets (including the null set and S itself) of S. Show that

$$\mathbb{P}(A \subset B) = \left(\frac{3}{4}\right)^n.$$

Exercise 7. As a simplified model for weather forecasting, suppose that the weather (either wet or dry) tomorrow will be the same as the weather today with probability p. Show that the weather is dry on January 1, then p_n, the probability that it will be dry n days later, satisfies

$$p_n = \frac{1}{2} + \frac{1}{2}(2p - 1)^n.$$