Probability, homework 8, due April 1st.

Exercise 1. Find an example of real random variables $(X_n)_{n\geq 1}$, X, in L^1, such that $(X_n)_{n\geq 1}$ converges to X in distribution and $\mathbb{E}(X_n)$ converges, but not towards $\mathbb{E}(X)$.

Exercise 2. Let $(X_n)_{n\geq 1}$, $(Y_n)_{n\geq 1}$ be real random variables, with X_n and Y_n independent for any $n \geq 1$, and assume that X_n converges in distribution to X and Y_n to Y. Prove that $X_n + Y_n$ converges in distribution to $X + Y$.

Exercise 3. Let X and Y be independent random variables uniform on $[0, 1]$. What is $\mathbb{E}(|X - Y|)$?

Exercise 4. Prove that if a sequence of real random variables (X_n) converge in distribution to X, and (Y_n) converges in distribution to a constant c, then $X_n + Y_n$ converges in distribution to $X + c$.

Exercise 5. Let $(X_n)_{n\geq 1}$ be a sequence of i.i.d. random variables with standard Cauchy distribution, on the same probability space, and let $M_n = \max(X_1, \dots, X_n)$. Prove that $(nM_n^{-1})_{n\geq 1}$ converges in distribution and identify the limit.

Exercise 6. Let $(X_n)_{n\geq 1}$ be a sequence of independent real random variables, all uniformly distributed on $[0, 1]$. Does $n \inf(X_1, \dots, X_n)$ converge in law as $n \to \infty$? If yes, what is the limiting distribution?