Problems:

1. There are \( \binom{6}{3} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20 \) possible samples. We list them here:

   \begin{align*}
   & abc \quad abd \quad abe \quad abf \\
   & acd \quad ace \quad acf \quad ade \\
   & adf \quad aef \quad bcd \quad bce \\
   & bcf \quad bde \quad bfd \quad bef \\
   & cde \quad cdf \quad cef \quad def
   \end{align*}

   If we assume random sampling, we assume that each of these are equally probably, and thus have a probability of \( 1/20 = .05 \).

2. There are \( \binom{6}{2} = \frac{6 \times 5}{2 \times 1} = 15 \) possible samples. We list them here:

   \begin{align*}
   \text{sample} & \quad \text{sample mean} \\
   110, 115 & \quad 112.5 \\
   110, 140 & \quad 125 \\
   110, 145 & \quad 122.5 \\
   110, 160 & \quad 145 \\
   110, 195 & \quad 152.5 \\
   115, 140 & \quad 127.5 \\
   115, 145 & \quad 130 \\
   115, 160 & \quad 137.5 \\
   115, 195 & \quad 155 \\
   140, 145 & \quad 142.5 \\
   140, 160 & \quad 150 \\
   140, 195 & \quad 167.5 \\
   145, 160 & \quad 152.5 \\
   145, 195 & \quad 170 \\
   160, 195 & \quad 177.5
   \end{align*}

   Since there are 15 possibilities, and we are assuming random sampling, then each of these samples has a probability of 1/15 of being chosen.

3. To compute the mean, we multiply each of these numbers by 1/15 and add, or, more simply, add the numbers up and divide by 15. We can also compute the mean of the original data, since we know these are the same. In any case, we obtain 144.17.

4. Recall that if \( \sigma \) is the standard deviation of the population, then the standard deviation of a sample of size \( n \) is given by \( \sigma/\sqrt{n} \). Changing the sample size from 10 to 100 changes the standard deviation from

   \[ \frac{\sigma}{\sqrt{10}} \rightarrow \frac{\sigma}{\sqrt{100}} \]
which decreases it by a factor of $\sqrt{10}$, or multiplies it by 0.3162. Changing the sample size from 250 to 25 changes the standard deviation from 

$$\frac{\sigma}{\sqrt{250}} \rightarrow \frac{\sigma}{\sqrt{25}}$$

which increases it by a factor of $\sqrt{10} = 3.162$.

5. Now the formula for the standard deviation of the sample mean is 

$$\frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}.$$

So, in the first case, the standard deviation changes from 

$$\frac{\sigma}{\sqrt{10}} \sqrt{\frac{990}{999}} = 0.3418\sigma$$

to 

$$\frac{\sigma}{\sqrt{100}} \sqrt{\frac{900}{999}} = 0.0949\sigma,$$

so that it is multiplied by a factor of 0.3015.

In the second case, we go from 

$$\frac{\sigma}{\sqrt{250}} \sqrt{\frac{750}{999}} = 0.0548$$

to 

$$\frac{\sigma}{\sqrt{25}} \sqrt{\frac{975}{999}} = 0.198\sigma,$$

so that it is multiplied by a factor of 3.606.

6. We replace the standard deviation of the population with that of the sample (we can do this since $n > 30$). Then we have that the standard deviation of the sample mean is $\sigma/\sqrt{40} = 19.45$. So, we have a maximum error of $E = 19.45z_{\alpha/2}$ with a confidence error of $1 - \alpha$.

To get a 95% confidence, we choose $\alpha = 0.05$ and that means we need to compute $z_{0.025}$. Recall that $z_{0.025}$ is the point at which the area to the right of this point under the standard normal distribution is 0.025, or, equivalently, the point so that the area from zero to that point is 0.475. Doing a reverse lookup in Table I gives $z_{0.025} = 1.96$. Thus our maximum error is $1.96 \times 19.45 = 38.122$.

Summarizing, we can say with 95% confidence that the real mean lies between 1126 - 38.122 ≈ 1087 and 1126 + 38.122 ≈ 1164.

7. This problem is the same except that we choose $\alpha = 0.01$, and we are looking for $z_{0.005}$. Doing a reverse lookup in Table I for 0.495 gives $z_{0.005} \approx 2.57$. Thus our maximum error is $2.57 \times 19.45 = 49.97$, roughly 50. Thus we can be 99% certain that the mean lies between 1076 and 1176.

8. This problem is similar to the previous two. We have a standard deviation of 3.21 with a sample size of 100, so our error is 

$$\frac{z_{0.025} \times 3.21}{\sqrt{100}} = \frac{1.96 \times 3.21}{10} = .629.$$

Thus our confidence interval is (4.231, 5.489). We can be 95% certain the mean value of pollution lies in this interval.