HARMONIC ANALYSIS AND SIGNAL PROCESSING SEMINAR

Discrete Sampling

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Courant Institute, Warren Weaver Hall 1314 *Wednesday, Feburary 24,* *2:00 pm*

Abstract

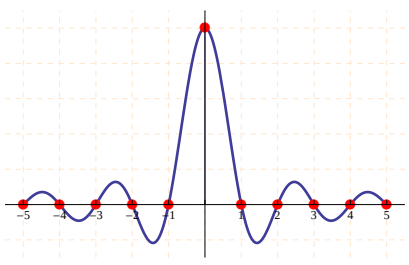
The Nyquist-Shannon Sampling Theorem, well-known to all electrical engineers, is used today in practically all telecommunications systems, control systems, audio electronics, imaging devices, and speech applications. We reexamine the Nyquist-Shannon Sampling Theorem in the discrete, finite dimensional setting, and ask when it is possible to interpolate a discrete signal from a limited number of samples. What vector spaces of signals will admit such an interpolation equation? What algebraic properties must the spaces and sampling patterns satisfy? The first part of the talk establishes a simple theoretical foundation for addressing this natural question.

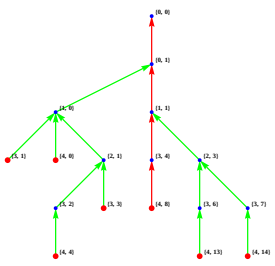
In the second part of the talk, we use this foundation to investigate discrete bandlimited spaces, in which certain frequencies are activated, and others are not. We present algorithms for constructing all possible Nyquist-Shannon-style interpolation equations for a given bandlimited space, yielding a sampling dictionary. Many patterns and symmetries are revealed in these dictionaries, establishing interesting connections between our work and group theory, number theory, and graph theory, specifically: orbit counting; prime factorization; perfect graphs and cliques; difference sets; vanishing sums of roots of unity and the DFT.

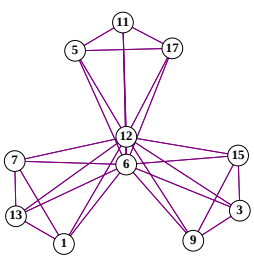
In the third part of the talk, we conduct a similar investigation of wavelet spaces, in which certain "sequencies" are activated. The problem of constructing interpolation equations then reduces to an analysis of posets and Hasse diagrams, yielding a connection with combinatorics.

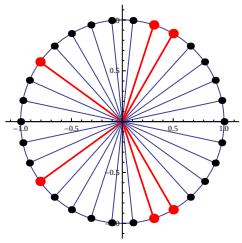
We conclude with open problems and ideas for future research.

This is joint work with Brad Osgood.[2]









[1]Stanford University, Electrical Engineering Department, Information Systems Laboratory
[2]Ibid.