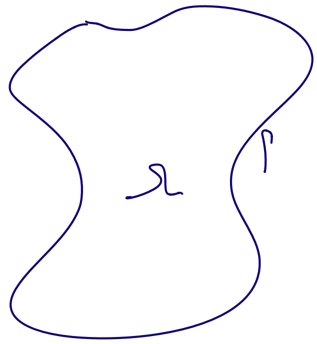
Comments on Discretization

$$\int_\Gamma G(\vec{x},\vec{y})\,\sigma(\vec{y})\,ds(\vec{y}) = f(\vec{x}).$$



① How to represent σ.

② Where to enforce the equation.

① **Option A** Use point values of σ

Option B Use an expansion: $\sigma(\vec{y}) = \sum_{j=1}^{k} \alpha_j \varphi_j(\vec{y})$.

basis functions.

② **Option 1** Enforce at a collection of $\vec{x}_i$'s.

Option 2 Enforce weakly: moments of the equation.

$$\Rightarrow \left(\psi_i, \int G\sigma\right) = (\psi_i, f)$$

$$(f,g) = \int_\Gamma f(s)\,\overline{g(s)}\,ds$$

A1 : Nyström $\quad A_{ij} = w_{ij}\, G(\vec{x}_i, \vec{x}_j) \qquad f_i = f(\vec{x}_i)$.

B1 : Collocation $\quad A_{ij} = \int_\Gamma G(\vec{x}_i, \vec{y})\,\varphi_j(\vec{y})\,ds(\vec{y}) \qquad f_i = f(\vec{x}_i)$

A2 : Qualocation $\quad A_{ij} = \int_\Gamma \psi_i(\vec{x})\, G(\vec{x}, \vec{y}_j)\,ds(\vec{x}), \quad f_i = (\psi_i, f)$

BZ: Galerkin $\qquad A_{ij} = \int_\Gamma \int_\Gamma \varphi_i(\vec{x})\, G(\vec{x},\vec{y})\, \varphi_j(\vec{y})\, ds(\vec{y})\, ds(\vec{x})$

$$f_i = (\varphi_i, f)$$

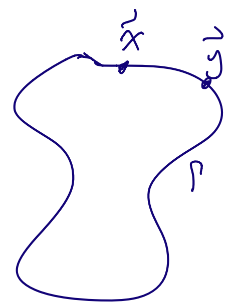
$$\Delta u = f$$

Weak: $\int v_i \,\Delta u_j = \int v_i f$

$$\sim \int \nabla v_i \cdot \nabla u_j = \int v_i f$$

Generality of log

$$\int_\Gamma \log|\vec{x}-\vec{y}|\; \sigma(\vec{y})\; ds(\vec{y})$$



$$= \int_0^1 \log|\vec{x}(t)-\vec{x}(t')|\; \sigma(\vec{x}(t'))\; |\vec{x}'(t)|\; dt' \qquad \Gamma: t \in [0,1] \to \vec{x}(t) \in \mathbb{R}^2$$

$$\frac{t-t'}{t-t'}$$

$$= \int_0^1 \log\left|\frac{\vec{x}(t)-\vec{x}(t')}{t-t'}(t-t')\right| \;\sigma\; |\vec{x}'|\; dt'$$

$$= \int_0^1 \left(\overbrace{\log \left| \underbrace{\frac{\vec{x}(t) - \vec{x}(t')}{t - t'}}_{\approx |\vec{x}'(t)| \text{ when } t' \to t} \right|}^{\text{not singular}} \sigma + \underbrace{\log|t-t'| \, \sigma(t')}_{\text{does not depend on the parameterization.}} \right) |\vec{x}'(t')| \, dt'$$

$\Rightarrow$ Quadratures for $\int \log|s-t| \, p(t) \, dt$ can be used for $\int_\Gamma \log|\vec{x}-\vec{y}| \, \sigma(\vec{y}) \, ds(\vec{y})$.

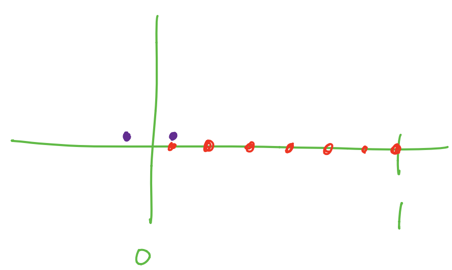
No such trick exists for surface integrals of the form

$$\int_\Gamma \frac{\sigma(\vec{y})}{|\vec{x} - \vec{y}|} \, da(\vec{y})$$

where Γ is a surface embedded in 3D.

<u>Comment on Kapur - Rokhlin</u>

$$\int_0^1 \log|x| \, p(x) \, dx$$



Apply the trapezoidal rule:

$$\approx h \sum_{j=1}^{N} \log|jh| \, p(jh) - \log 1 \cdot p(1) \qquad h = \frac{1}{N}$$

Add in corrections

$$\approx a\big(p(-h) + p(h)\big) + T_N(\log p).$$

Solve for a such that this is exact for $p =$ const.