Theory of Probability

Sep 28, 2020

Recall that

$$P(E|F) = \frac{P(EF)}{P(F)}.$$

If F doesn't affect $E|F$, then

$P(E|F) = P(E)$ and we say that E, F are independent events.

This means that $P(E|F) = P(E) = \dfrac{P(EF)}{P(F)}$.

$$\Rightarrow P(EF) = P(E)\,P(F).$$

Independence.

Definition: E and F are independent events if $P(EF) = P(E)\,P(F)$.

If $P(EF) \neq P(E)\,P(F)$ then E, F are dependent events.

Example Roll 2 dice:

E = sum is 6
F = die 1 is 4.

$P(E) = 5/36$

$P(F) = 1/6$

$$P(EF) = P\begin{pmatrix} \text{die1}=4 \\ \text{die2}=2 \end{pmatrix} = \frac{1}{36}$$

$$P(E)\,P(F) = \frac{5}{36}\,\frac{1}{6} = \frac{5}{216} \neq P(EF) = 1/36.$$

$\Rightarrow E, F$ are not independent. (they are dependent.)

Example $G =$ sum $= 7$
$F =$ die 1 is 4

$$P(F) = 1/6 \qquad P(FG) = P\begin{pmatrix} \text{die } 1 = 4 \\ \text{die } 2 = 3 \end{pmatrix} = 1/36$$

$$P(G) = 6/36 = 1/6$$

$$P(F)P(G) = 1/36$$

$\Rightarrow P(FG) = P(F)\,P(G) \Rightarrow F, G$ are independent.

We can also say:

Proposition 4.1: If E, F are independent, then so are E and F^c.

$$P(E) = P(EF) + P(EF^c)$$
$$= P(E)P(F) + P(EF^c)$$

$$\Rightarrow P(EF^c) = P(E) - P(E)P(F)$$
$$= P(E)(1 - P(F))$$
$$= P(E)P(F^c)$$

✓ E, F^c are independent.

Question: If E,F are independent, and E,G are independent, is E independent of the event FG?

Example: Roll 2 dice

$E =$ sum is 7
$F =$ first die is 4
$G =$ second die is 3.

From the previous example, $P(EF) = P(E)P(F)$.

$P(EG) = P(E)P(G)$.

$$P(E \mid FG) = \underline{1} \neq P(E) = 1/6.$$

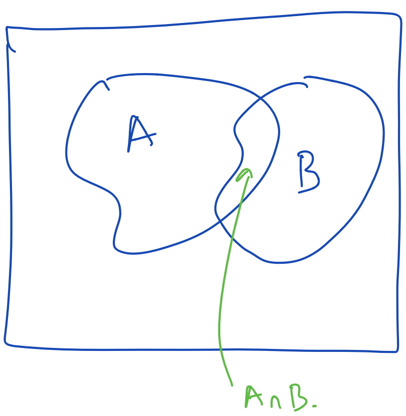
<u>Definition</u> The events E, F, G are independent if

$$P(EFG) = P(E)P(F)P(G)$$
$$P(EF) = P(E)P(F)$$
$$P(EG) = P(E)P(G)$$
$$P(FG) = P(F)P(G).$$

If these conditions hold, then E is independent of any function of F, G. (I.E. E and FG are independent.)

<u>Example</u>: Toss a coin N times, the outcome of each toss is H, T and independent of all other tosses.

Each toss in this example is known as a "trial".

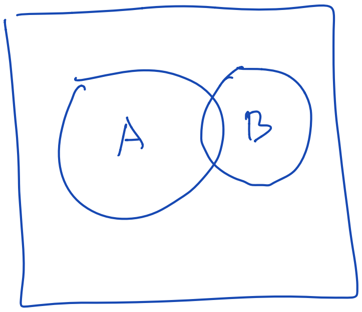


$P(A \mid B) = P(A)$ (if independent).

$= \dfrac{P(AB)}{P(B)}$ = the ratio of the area in AB to the area of B (If events are drawn to scale.)

$P(A)$ = ratio of the area of A to the whole sample space ($P(S) = 1$)

Example



are these independent? Don't know...