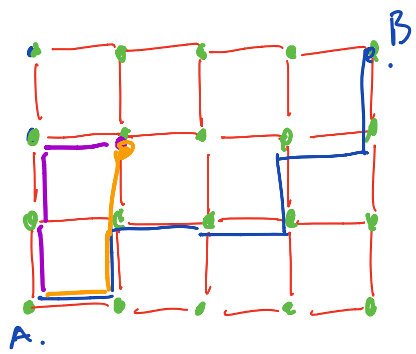
Theory of Probability Sep 9, 2020

Permutations : counting the possible orderings of distinct objects

Combinations : counting the groupings of objects when order doesn't matter.

Example :



Can only move up or right.

Question: How many such paths are there?

Path 1: U U R

Path 2 : R U U

From A → B: 4 rights R
3 ups. U

7 moves in total. Any path : U R U R R U R

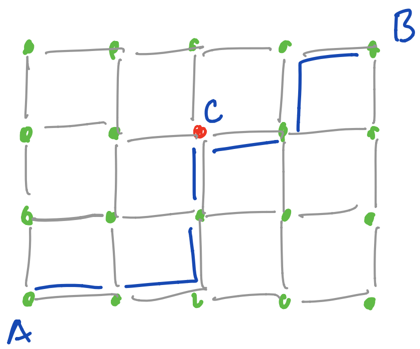
(I) $\binom{7}{4} = \frac{7!}{4!\,3!} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 7 \cdot 5 = 35$

(II) $\frac{7!}{4!\,3!}$ permutations

#permutations of U's

#permutations of R's

A related problem:



Question: How many paths go through node C?

M paths from A → C
N paths from C → B,
then MN paths pass through C.

$$M = \binom{4}{2} = \frac{4 \cdot 3}{2} = 6$$

$$N = \binom{3}{2} = \frac{3 \cdot 2}{2} = 3 \quad \Rightarrow \quad MN = 18$$

Note $\binom{N}{k} = \binom{N}{N-k}$ $\quad \binom{3}{2} = \binom{3}{1}$

$$\frac{N!}{k!(N-k)!}$$