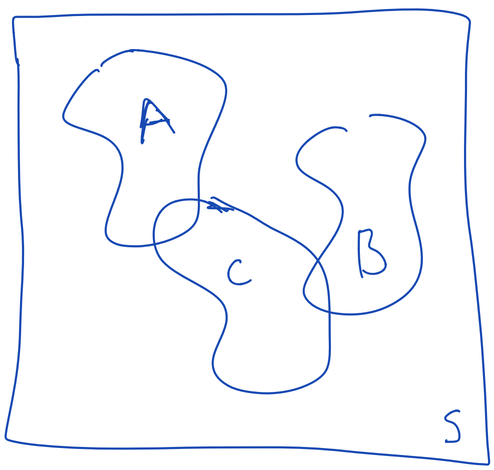
Theory of Probability

Sep 16, 2020

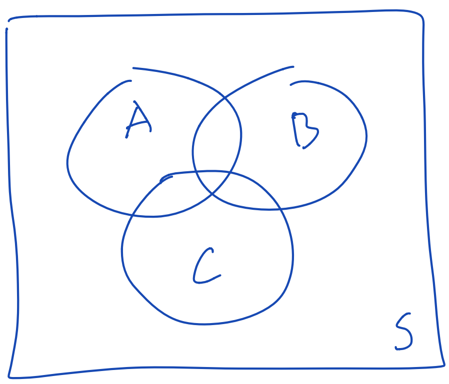
Prop. 4.4 : Inclusion-Exclusion



$$P(A \cup C) = P(A) + P(C) - P(A \cap C)$$

$$P(S) = 1$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap C) - \underbrace{\cancel{P(A \cap B)}}_{=0} - P(B \cap C) + \underbrace{\cancel{P(A \cap B \cap C)}}_{=0}$$



Problem 9 from Ch. 2:

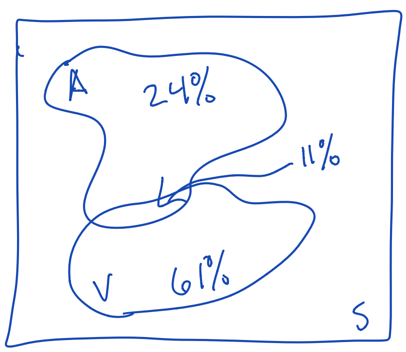
24% carry AMEX

61% carry VISA

11% carry both cards.

[What percent carry AMEX <u>OR</u> VISA?

$24\% + 61\% - 11\% = 74\%$



What percent carrys Amex but <u>not</u> Visa? 13%

Theoretical Exercise 11

Let $P(E) = 0.9$

$P(F) = 0.8$

Show that $P(E \cap F) = P(EF) \geq .7.$

By Inclusion-Exclusion,

$$P(E \cup F) = \underline{P(E)} + \underline{P(F)} - \underline{P(EF)}$$

$$= 0.9 + 0.8 - P(EF)$$

$$P(E \cup F) = 1.7 - P(EF)$$

Since $P(E \cup F) \leq 1$, it must be that $P(EF) \geq .7$

More generally, we have Bonferroni's Inequality:

$$P(EF) \geq P(E) + P(F) - 1$$

$$P(E \cup F) = P(E) + P(F) - P(EF)$$

Since $P(E \cup F) \leq 1$ we have that

$$P(E) + P(F) - P(EF) \leq 1$$

$\Rightarrow$

$$P(EF) \geq P(E) + P(F) - 1.$$

Problem 7 from Ch. 2

15 members of a soccer team, each is either blue collar / white collar, and either R/D/I.

How many outcomes are

(a) in the sample space?

6^{15}

Person 1	Person 2	Person 3 ...
B R		
B D		
B I		
W R		
W D		
W I		

(b) How many outcomes are in the event that at least one of the team members is a blue collar worker?

Complement: None of the team members are blue collar workers.

$6^{15} - \underbrace{\#\text{ outcomes when none are blue collar.}}_{3^{15}}$

(c) How many outcomes are in the event that none of the team members consider themselves Independent?

4^{15}