Theory of Probability
$$(X \sim Poisson(\lambda))$$
. Oct 19,2020
 $P[X = k] = e^{-\lambda} \frac{\lambda^k}{k!}$ when $\lambda > 0$ is some parameter.

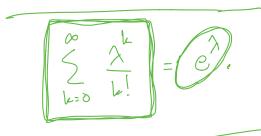
$$E[X] = \lambda$$

$$Var[X] = \lambda.$$

If
$$Y \sim Binomial(n,p)$$
 random variable, and if n is large and p is small, with $np = \lambda \sim O(1)$, then

Y is approximately Poisson (2).

$$P[Y=L] \approx P[X=L]$$
 when $X \sim Poisson(\lambda)$.



Theortical Exercin 4.17

X~ Poisson(x).

Show P[X=k] increases monotonically, then decreases monotonically as a faction of k.

(1) Show that
$$\frac{P[X=k+1]}{P[X=k]}$$
 >1

$$\frac{P[X=k+1]}{P[X=k]} = \frac{2}{(k+1)^{\frac{1}{2}}} = \frac{\lambda}{(k+1)^{\frac{1}{2}}} = \frac{\lambda}{k+1}$$

$$\frac{\lambda}{k+1} > 1 \quad \text{if} \quad k+1 < \lambda$$

$$= k < \lambda - 1$$

Self-test Exercise 4.14

On average, 5.2 horricanes hit a certain region every year.

What is the probability that there will be 3 or fewer hurricans this year?

let X = # of harrianes this year, then

 $X \sim Poisson(\lambda)$, and $\lambda = 5.2$.

X~ Poisson (xt), with t= 1 year, 2=5-2/year.

$$P[X \le 3] = P[X = 0] + P[X = 1] + P[X = 2] + P[X = 3]$$

$$= \sum_{k=0}^{3} e^{-\lambda} \frac{\lambda^{k}}{k!} = \approx 24\%$$

Self-test exercise 4.15

$$X \approx Possion(X)$$
 which models the number of eggs laid on a leaf by a certain insect.

Let $Y = positive value of X$
 $P[Y = k] = P[X = k | X > 0]$

What is $E[Y]$.

 $E[Y] = \sum_{k=1}^{\infty} k P[Y = k]$
 $= \sum_{k=1}^{\infty} k P[X = k | X > 0]$
 $= \frac{2}{k} k P[X = k] X > 0$
 $= \frac{2}{k} k P[X = k]$
 $= \frac{1}{1-e^{2}} \sum_{k=0}^{\infty} k P[X = k]$