Theory of Probability

Oct 26, 2020

Continuous R.V.'s are completely characterized by their probability density function:

If X is a continuous random variable, then

$$P[X \in [a,b]] = \int_a^b f(x)\,dx$$

density function.

$$\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 1$$

The cumulative distribution function is defined analogously:

$$P[X \le x] = \int_{-\infty}^{x} f(t)\,dt = F(x).$$

Fundamental theorem of Calculus says:

$$= \frac{d}{dx}\int_{-\infty}^{x} f(t)\,dt = f(x) = \frac{d}{dx}F(x) = F'(x)$$

$$F'(x) = f(x).$$

$$\int_a^x F'(t)\,dt = F(x) - F(a)$$

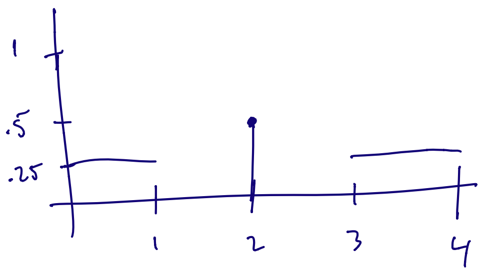
$$\int_{-\infty}^{x} F'(t)\,dt = F(x) - F(-\infty)$$

Expectation :

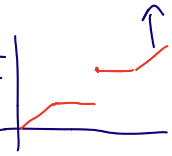
$$E[X] = \int x\, f(x)\, dx = \mu$$

Variance :

$$Var[X] = \int (x-\mu)^2 f(x)\, dx = \sigma^2$$

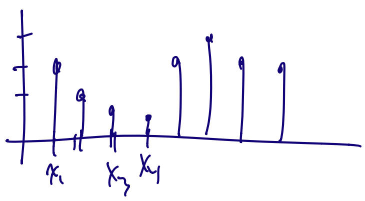


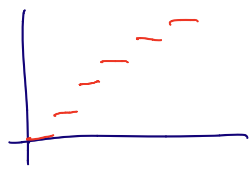
$P[X=2] = .5$

F 

$$= \int_2^2 f(x)\, dx = 0$$

$$= \lim_{\epsilon \to 0} \int_{2-\epsilon}^{2+\epsilon} f(x)\, dx = .5$$



F : 

Theo exercise 5.2

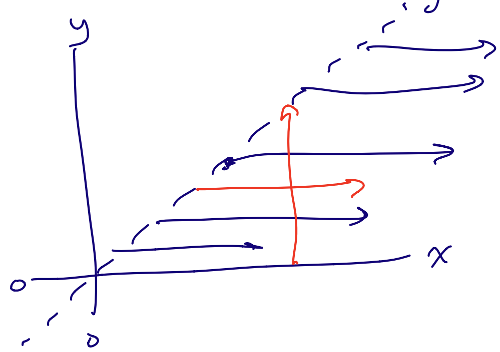
Show that $E[Y] = \int_0^\infty P[Y > y]\, dy - \int_0^\infty P[Y < -y]\, dy$

Let f be the density function of Y.

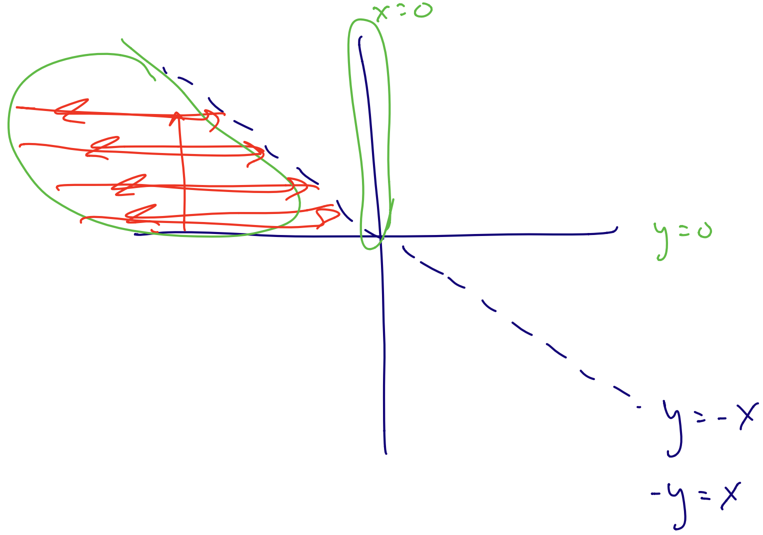
$$\int_0^\infty \int_y^\infty f(x)\, dx\, dy$$

$$= \int_0^\infty \int_0^x f(x)\, dy\, dx$$

$$= \int_0^\infty f(x) \int_0^x dy\, dx = \int_0^\infty x\, f(x)\, dx$$



$$\int_0^\infty P[Y < -y]\, dy$$



$$= \int_0^\infty \int_{-\infty}^{-y} f(x)\, dx\, dy$$

$$= \int_{-\infty}^{0} \int_0^{-x} f(x)\, dy\, dx$$

$$= \int_{-\infty}^{0} f(x) \int_0^{-x} dy\, dx \quad = \int_{-\infty}^{0} f(x) \left[y \right]_0^{-x} dx$$

$$= \int_{-\infty}^{0} -x\, f(x)\, dx$$

$$= -\int_{-\infty}^{0} x\, f(x)\, dx$$

$$\int_0^\infty P[Y > y]\, dy - \int_0^\infty P[Y < -y]\, dy = \int_0^\infty x\, f(x)\, dx - \left(- \int_{-\infty}^{0} x\, f(x)\, dx \right)$$

$$= E[Y].$$