Theory of Probability

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Poisson Process:

$N(t)$ = number of events occuring in the time interval $[0,t]$

In a small window of time $[0,h]$,

$$P[N(h)=1] = \lambda h + o(h)$$

$$P[N(h)\geq 2] = o(h).$$

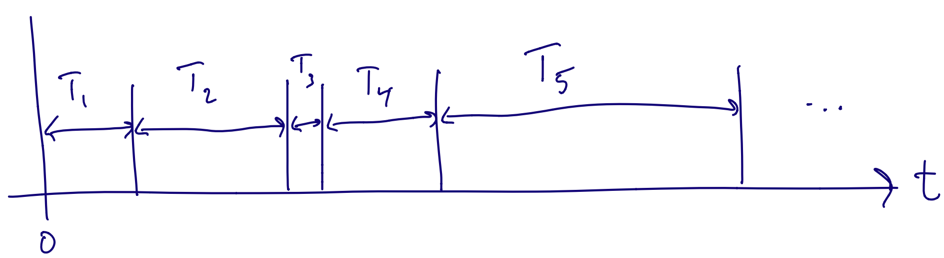
$$\Rightarrow \quad P[N(t)=0] = e^{-\lambda t}$$

T_n = n^{th} interarrival time

= time between event n and $n-1$

$$T_n \sim Exp(\lambda)$$

Intuition



Time of n^{th} event:

$$S_n = \sum_{i=1}^{n} T_i$$

$$\sim Gamma(n, \lambda).$$

$$\Rightarrow \quad P[N(t) = n] = \frac{e^{-\lambda t} (\lambda t)^n}{n!}$$

$$\Rightarrow \quad N(t) \sim \text{Poisson}(\lambda t)$$

Final Exam Topics

Ross: 6.1 → 6.7
7.1–7.2, 7.4 → 7.8
8.1 → 8.5.

Ch 6. Jointly distributed random variables:

- joint pdfs
  - → marginal distribution
  - → conditional
- independence
- sums of random variables
  - → convolutions of pdfs.
- order statistics
- functions of multiple random variables.
  - ↳ If $Y_1 = g_1(X_1, X_2)$
    $Y_2 = g_2(X_1, X_2)$

    what is $f_Y(y_1, y_2)$?

Ch. 7: Expectation

- Expectation is a linear function
  $$E[aX + bY + c] = aE[X] + bE[Y] + c.$$
- Covariance / correlation
- Conditional Expectations
  - $E[E[X \mid Y]] = E[X]$.
- Moment generating functions
- Multivariate Normal Random variable

If $\begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix}$ is a multivariate normal random variable, it is completely characterized by the covariances $Cov(X_i, X_j) = \Sigma_{ij}$ and $E[X_i] = \mu_i$

Ch 8: Limit Theorems

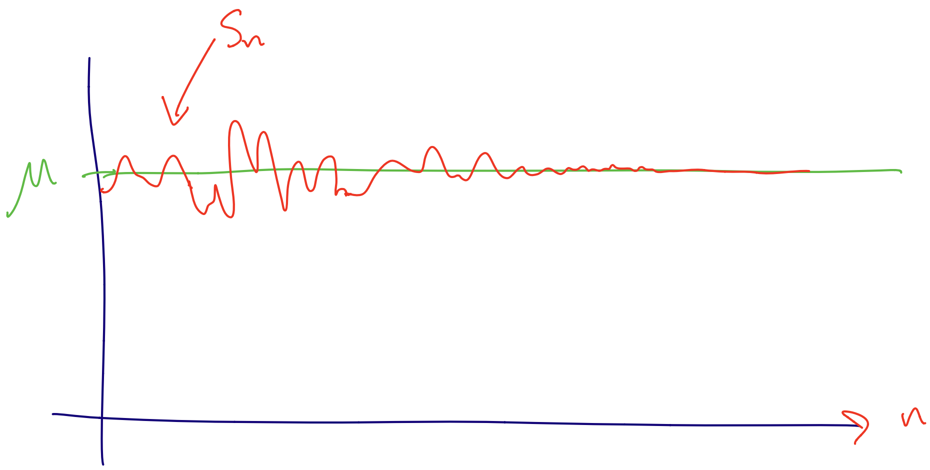
- Know how to use Markov, Chebyshev, and Jensen's Inequalities.
- SLLN and WLLN
- Central Limit Theorem

SLLN

$$P\Big[\underbrace{\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} X_i}_{S_n} = \mu\Big] = 1$$

where $E[X_i] = \mu < \infty$.

$(E[X_i^4] < \infty)$



WLLN

For any $\epsilon > 0$, $\lim_{n\to\infty} P\Big[\Big|\frac{1}{n}\sum_{i=1}^{n} X_i - \mu\Big| \geq \epsilon\Big] = 0$

$(\mathrm{Var}\, X_i < \infty)$.