Notes Preparation 9.21

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1 Section 2.5: Sample Space Having Equally Likely Outcomes

1.1 Introduction

In many experiments, it is natural to assume that all outcomes in the sample space are equally likely to occur. That is, consider an experiment whose sample space S is a finite set, say, $S = 1, 2, ..., N$. Then it is often natural to assume that P(1) = P(2) = · · · = P(N). From above, we say P(i) = 1/N (i = 1,2,...,N).

1.2 Definition

Put the conclusion that we got in 1.1 into more general cases is that if we assume that all outcomes of an experiment are equally likely to occur, then the probability of any event E equals the proportion of outcomes in the sample space that are contained in E.
P(E)= (the number of outcomes in E) / (the number of outcomes in S)

1.3 Examples

Problem. If two dice are rolled, what is the probability that the sum of the upturned faces will equal 7?

There are total 6*6= 36possible outcomes which are equally likely. Since there are 6 possible outcomes—namely, (1, 6), (2,5), (3, 4), (4, 3), (5, 2), and (6, 1)—that result in the sum of the dice being equal to 7, the desired probability is 6/36 = 1/6 .

Ex2:

If 3 balls are "randomly drawn" from a bowl containing 6 white and 5 black balls, what is the probability that one of the balls is white and the other two black?

RMK: We can solve this problem in two ways. In general, When the experiment consists of a random selection of k items from a set of n items, we have the flexibility of either letting the outcome of the experiment be the ordered selection of the k items or letting it be the unordered set of items selected.

Ans:

1)Ordered: The sample space consists of $11 \cdot 10 \cdot 9 = 990$ outcomes. There are $6{\cdot}5{\cdot}4 = 120$ outcomes in which the first ball selected is white and the other two are black; similar for the other two cases. Therefore, accroding to the definition, the probability is $(120{+}120{+}120)/990 = 4/11$.

2)Unordered: From this point of view, there are

$$\binom{11}{3} = 165$$

outcomes in the sample space. Now, each set of 3 balls corresponds to 3! outcomes when the order of selection is noted. As a result, if all outcomes are assumed equally likely when the order of selection is noted, the probability is

$$\frac{\binom{6}{1}\binom{5}{2}}{\binom{11}{3}} = 4/11$$

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2 Section 2.7: Probability as a Measure of Belief

2.1 Introduction

Up until this point, our interpretation of the word "probability" has been exclusively linked to the chance that that a particular result of an experiment will take place if the experiment is repeated many times. However, we've seen the word "probability" used in other ways; namely, in situations of belief. For example, consider the statements, "There is a 90 percent probability that Shakespeare actually wrote *Hamlet*, or "The probability that Oswald acted alone in assassinating Kennedy is .8. How do we interpret these statements?

The most simple and natural interpretation is that the probabilities referred to are measures of the individual's degree of belief in the statements that they are making. In other words, the individual who made the statements is very sure that Oswald acted alone, and even more sure that Shakespeare wrote *Hamlet*. The idea of probability as a measure of the degree of one's belief in a statement is known as the *personal* or *subjective* view of probability. Consider the following problem.

Problem. Suppose that in a 7-horse race, you believe that each of the first 2 horses has a 20 percent chance of winning, horses 3 and 4 have a 15 percent chance, and the remaining 3 horses have a 10 percent chance each. Would it be better for you to wager at even money that the winner will be one of the first three horses or to wager, again at even money, that the winner will be one of the horses 1, 5, 6, and 7?

Solution. On the basis of your personal probabilities concerning the outcome of the race, your probability of winning the first bet is $.2 + .2 + .15 = .55$, whereas it is $.2 + .1 + .1 + .1 = .5$ for the second bet. Hence, the first wager is more attractive.

Note that when we assume that a person's subjective probabilities are perfectly consistent with the axioms of probability, we are considering an idealized, rather than realistic, person. For example, if we were to ask someone what they thought the chances were of
(a) rain today,
(b) rain tomorrow,
(c) rain both today and tomorrow,
(c) rain either today or tomorrow,
it's quite possible that, after some deliberation, they might answer 30 percent, 40 percent, 20 percent, and 60 percent as answers. However, note that these answers are not consistent with the axioms of probability.