Homework 1

Due: 2:00pm Feb. 4th, 2016

Each problem is worth 10 points.

Exercise 1 [Newton's method]: On your computer, it is easier to build hardware to perform multiplication than it is for division (multiplying is easier than long division!). Therefore, it would be nice to compute $a = b/c$ by *multiplying* the number b by the number $1/c$. This first requires that we compute $1/c$, where we know what c is. Derive a formula using Newton's method that allows you compute $1/c$ without using division – only multiplication and addition/subtraction are allowed.

Exercise 2: How would you find all the roots of the function $f(x) = \cos x + 1$? Can you use bisection? Secant? Newton? After choosing a method, determine the rate of convergence of the method.

Exercise 3: If you enter a number in your calculator and repeatedly push cos, it will converge to a particular number. What is this number? Provide a proof!

Exercise 4: Let $0 < \epsilon < 1$ and $a \in [0, \pi]$ be known numbers. Consider the equation

$$y - \epsilon \sin y = a.$$

Write this equation in fixed-point form, and show that it has a unique solution.

Hint: Show that $|\sin y - \sin x| \leq |y - x|$ for all x, y by using Taylor's Theorem with a remainder to express $\sin y$ as $\sin x$ plus a remainder term.