Homework 10
Due: 2:00pm April 21, 2016

Each problem is worth 10 points.

Exercise 1 : Which of the following multistep methods are *convergent*? You will not receive any credit unless you justify your answer:

1. $y_k - y_{k-2} = h(f_k - 3f_{k-1} + 4f_{k-2})$

2. $y_k - 2y_{k-1} + y_{k-2} = h(f_k - f_{k-1})$

3. $y_k - y_{k-1} - y_{k-2} = h(f_k - f_{k-1})$

Exercise 2 : Consider the system of equations

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} -1000 & 1 \\ 0 & -1/10 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$
$$x(0) = 1, \qquad y(0) = 2.$$

Using the fourth-order Runge-Kutta method, what is the maximum step-size h allowed for stability reasons?

Exercise 3 : Consider the 10×10 matrix **A**:

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & -1 \\ & & & -1 & 2 \end{bmatrix}.$$

Where are the eigenvalues of this matrix located?