Homework 8

Due: 2:00pm April 7, 2016

Each problem is worth 10 points.

Exercise 1 : Solve the following initial value problems analytically:

1. $y' = t^3$, $y(0) = 0$
2. $y' = 2y$, $y(1) = 3$
3. $y' = ay + b$, $y(0) = y_0$ (Assume that a and b are scalars. Hint: Multiply by the integrating factor e^{-ta} and integrate from 0 to T)

Exercise 2 : Verify that the function $y(t) = t^{3/2}$ solve the initial value problem

$$y' = \frac{3}{2} y^{1/3}, \qquad y(0) = 0.$$

Apply Euler's method to this problem and explain why the numerical approximation differs from the solution $t^{3/2}$.

Exercise 3 : Write down the result of applying one step of Euler's method to the initial value problem $y' = (t + 1)e^{-y}$, $y(0) = 0$, using step size $h = 0.1$. Do the same for the midpoint method and for Heun's method.

Exercise 4: Write down the result of applying one step of Euler's method to the predator-prey equations:

$$R'(t) = (2 - F(t))\, R(t)$$
$$F'(t) = (R(t) - 2)\, F(t),$$

starting with $R(0) = 2$ and $F(0) = 1$ and using step size $h = 0.1$. Do the same for the midpoint method and for Heun's method.