Lecture 13 Numerical Analysis 03/20/18

Notes on the midterm:

Range: 20 – 48

High grade: 48

Average: 34

std: 7

Comments on specific problems:

1a 2c 4 5a 5d

Back to the course...

Last time: Least squares, Gram-Schmidt, the SVD

The next class of algorithms is for computing eigenvalues and eigenvectors.

2 Motivating Applications:

<u>Differential equations</u>:

$$\frac{d\underline{x}(t)}{dt} = A\,\underline{x}(t) \qquad \underline{x}(t) = \begin{pmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{pmatrix}$$

time-independent matrix

If $A = PDP^{-1}$, $P = \begin{pmatrix} \underline{u}_1 & \cdots & \underline{u}_n \end{pmatrix}$ eigenvectors

$D = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}$ eigenvalues

Then

$$\frac{d\underline{x}(t)}{dt} = PDP^{-1}\underline{x}(t)$$

$$\Rightarrow \quad \frac{d}{dt}\left(P^{-1}\underline{x}(t)\right) = D\left(P^{-1}\underline{x}(t)\right)$$

Let $\underline{y}(t) = P^{-1}\underline{x}(t)$

$$\Rightarrow \quad \frac{d\underline{y}(t)}{dt} = D\,\underline{y}(t)$$

$$\Leftrightarrow \quad \left.\begin{aligned} y_1'(t) &= \lambda_1 y_1(t) \\ &\vdots \\ y_n'(t) &= \lambda_n y_n(t) \end{aligned}\right\}$$

The equations have been decoupled.

Application 2: "The Google Matrix" and Pagerank

See:

Bryan and Leise, *The $25,000,000,000 Eigenvector: The Linear Algebra behind Google*, SIAM Review, Vol 48 No 3, pp. 569-581, 2006.