Homework 4

Due: 12:30pm April 5, 2018

Notes on the assignment:

Submission: Homework assignments must be submitted in hardcopy at the beginning of class on the due date. Late homework will not be accepted. You are encouraged to use the original homework LaTeX document as a template to write-up your homework. If you are required to hand in code, this will explicitly be stated on the homework assignment.

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1. (10 pts) Let $\boldsymbol{A}$ be an $n \times n$ real symmetric positive definite matrix. Prove that the solution to the system $\boldsymbol{Ax} = \boldsymbol{b}$ is the unique minimizer of the function

$$f(\boldsymbol{x}) = \frac{1}{2}(\boldsymbol{x}, \boldsymbol{Ax}) - (\boldsymbol{x}, \boldsymbol{b}).$$

2. (10 pts) Let the matrix $\boldsymbol{A}$ have eigenvalues $\lambda_1 > \lambda_2 > \ldots > \lambda_n$ (i.e. all real, but not necessarily positive). What shift should be used in the power method in order to make it converge most rapidly to λ_1 and $\boldsymbol{v}_1$, where $\boldsymbol{v}_1$ is the eigenvector corresponding to λ_1? And what shift should be used to make the power method converge most rapidly to λ_n and $\boldsymbol{v}_n$?

3. (10 pts) Let the eigenvalues of the $n \times n$ matrix $\boldsymbol{A}$ be 2, 4, 8, 16, ..., 2^n. If you are allowed to use the inverse power method with shifts $s = 2^\ell + 1$, for some ℓ, what is the fastest rate at which the scheme will converge to $\lambda_k = 2^k$ and $\boldsymbol{v}_k$?

4. (10 pts) Write a code that computes the eigenvalues of a symmetric matrix $\boldsymbol{A}$ using Jacobi's Method. Your algorithm should terminate when the square root of the sum of the squares of the off-diagonal elements of the transformed matrix is less than 10^{-8}. Apply your code to the matrices of dimensionalities 10, 20, and 40 with ij-entries given by

$$A_{ij} = \sqrt{i^2 + j^2}.$$

Your homework submission should include a list of eigenvalues in each case (showing 8 significant digits) and a printout of your code.