Homework 1

Due Tuesday, July 7th at the beginning of class

1. Fix $a_0, d \in \mathbb{R}$ and let $a_k = a_{k-1} + d$ for all $k \geq 1$. Show that $\frac{1}{n} \sum_{i=0}^{n-1} a_k$ is given by $(a_{n-1} + a_0)/2$.

2. Let $S = \{1, 2, \ldots, n\}$.
   (a) By counting the number of subsets of $S$ in two ways, prove that
   
   \[ 2^n = \sum_{k=0}^{n} \binom{n}{k}. \]
   (b) How many ways are there to choose 2 disjoint subsets $A, B$ from $S$? For example, if $n = 10$ then some distinct choices are:
   
   $A = \{1, 3\}$, $B = \{4\}$
   $A = \{4\}$, $B = \{1, 3\}$
   $A = \{1, 2, 3\}$, $B = \emptyset$
   $A = \emptyset$, $B = \emptyset$
   $A = \emptyset$, $B = \{1, 2, \ldots, 10\}$.

3. You have a team of 15 (distinguishable) players and must choose which 11 will play. There are two kinds of roles ($A, B$) for each player chosen. Seven will play role $A$, and four will play role $B$.
   (a) If 8 are able to play role $A$ and 7 are able to play role $B$, how many ways are there to choose who will play each role?
   (b) If 8 are able to play role $A$, 5 are able to play role $B$ and 2 can play either, how many ways are there to choose who will play each role?