Homework 2

Due Wednesday, July 8th at the beginning of class

1. Prove that if $A, B$ are events then

$$P(A \cap B) \geq P(A) + P(B) - 1.$$ 

*Solution.* Note that

$$1 \geq P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

By moving 1 to the right and $P(A \cap B)$ to the left we get the result.

2. You roll a fair 6-sided die and a fair 4-sided die.

(a) Give a sample space for the outcomes, and define $P$.

(b) Use your sample space to compute the probability of rolling a 3 on the first die.

(c) Use your sample space to compute the probability that the sum is 7.

(d) Suppose instead you rolled a fair 6-sided die, and then a fair $k$-sided die where $k$ is the value you got on your first roll. What is the sample space, what is $P$, and what is the chance of the sum being 7? (If you roll a 1-sided die, you always get 1.)

*Solution.*

(a) Let $S = \{(d_1, d_2) : 1 \leq d_1 \leq 6, 1 \leq d_2 \leq 4\}$ and

$$P(A) = \frac{|A|}{|S|} = \frac{|A|}{24}.$$ 

Alternatively, you could have said that

$$P(\{(d_1, d_2)\}) = \frac{1}{24}$$

for any $(d_1, d_2)$ and defined $P(A)$ as in our general finite space (end of lecture 2 notes).

(b) If $A$ denotes the event of rolling a 3 on the first die, we have

$$P(A) = \frac{4}{24} = \frac{1}{6}.$$
(c) There are 4 ways to get a sum of 7, so the probability is $\frac{1}{6}$.

(d) Here the sample space is

$$S = \{(d_1, d_2) : 1 \leq d_1 \leq 6, 1 \leq d_2 \leq d_1\}.$$  

We can treat this as a general finite space (see end of notes from lecture 2) with

$$P(\{(d_1, d_2)\}) = \frac{1}{6} \cdot \frac{1}{d_1}.$$  

If $A$ denotes the event the sum is 7 then we have

$$A = \{(4, 3), (5, 2), (6, 1)\}$$

so

$$P(A) = \frac{1}{6} \cdot \frac{1}{4} + \frac{1}{6} \cdot \frac{1}{5} + \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{6} \cdot \frac{74}{120} = \frac{37}{360}.$$