**Homework 9**

**Due Tuesday, July 21th at the beginning of class**

1. Suppose that a large lot with 10000 manufactured items has 30 percent defective items and 70 percent nondefective. You choose a subset of 10 items to test.

   (a) What is the probability that at most 1 of the 10 test items is defective?
   
   (b) Approximate the previous answer using the binomial distribution.

**Solution.**

(a) If \( X \sim \text{HGeom}(10, 3000, 10000) \) then we have

\[
p_X(0) + p_X(1) = \frac{\binom{7000}{10} + \binom{3000}{1} \binom{7000}{9}}{\binom{10000}{10}} \approx 0.149.
\]

(b) If \( Y \sim \text{Bin}(10, .3) \) we have

\[
p_Y(0) + p_Y(1) = (.7)^{10} + \binom{10}{1}(.3)(.7)^9 \approx 0.149.
\]

2. Fix \( 0 < p < 1 \) and suppose there is a coin that obtains a head with probability \( p \). We flip the coin 17 times and get a total of 5 heads. Given this information, what is the chance that 3 of those heads occurred in the first 10 flips?

**Solution.** Let \( F \) denote the event 3 occurred in the first 10 flips, and let \( E \) denote the event of getting 5 heads out of 17. Then we have

\[
P(F|E) = \frac{P(FE)}{P(E)} = \frac{\binom{10}{3} \binom{7}{5} p^5 (1-p)^{12}}{\binom{17}{5} p^5 (1-p)^{12}} = \frac{\binom{10}{3} \binom{7}{5}}{\binom{17}{5}}.
\]

In general, if \( X \sim \text{Bin}(n_1, p) \) and \( Y \sim \text{Bin}(n_2, p) \) are independent then

\[
P(X = k|X + Y = j) = \frac{\binom{n_1}{k} \binom{n_2}{j-k}}{\binom{n_1+n_2}{j}}.
\]

That is, conditional on the sum being \( j \) we get a \( \text{HGeom}(j, n_1, n_1 + n_2) \) distribution.