Simple Mathematical, Dynamical Stochastic Models Capturing the Observed Diversity of the El Niño Southern Oscillation (ENSO)

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Center for Atmosphere Ocean Science
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New York University

Graduate Course, MATH-GA.2830-001
Fall 2017, Courant Institute, NYU
Room 905, Thursdays 9:00–10:50am
Paradigm for Understanding the ENSO

Math Theory

Observations

Physical Phenomena
Simple Mathematical, Dynamical Stochastic Models Capturing the Observed Diversity of the El Niño Southern Oscillation

Andrew Majda

Room 905, Thursdays at 9-11am, 2017-2018 fall semester

The course should be interesting for undergraduate students, graduate students, and postdocs in pure and applied mathematics, physics, engineering, and climate, atmosphere, ocean science interested in modeling and simulating climate systems and other turbulent dynamical systems.
The El Niño-Southern Oscillation (ENSO) is the strongest signal on yearly time scale beyond the seasonal cycle. It has significant impact on global climate and relevance for seasonal forecasts. Roughly speaking, ENSO is a periodic phenomenon that people are familiar with. However, surprisingly the ENSO in the past few decades is very different from the traditional picture with large impacts on global warming and weather and climate around the world.

This course will focus on a new perspective on ENSO modeling, which is consistent with the recent observational record. Rigorous theories, simple nonlinear dynamics, simple stochastic models and numerical methods will be combined with physical reasoning to produce new simple nonlinear dynamics and the comparison of the model results with real observations will be emphasized. We will show how this can be done in a simple modeling framework that is consistent with complex observations of ENSO over the last a few decades.

This course is a self-contained introduction to these topics and elementary material from nonlinear dynamics, stochastic modeling and numerical algorithms will be developed in a self-contained way. Students and young researchers can certainly follow the lectures with elementary background.
The lectures will cover the basic physics of ENSO, real observational records and mathematical tools with interesting new nonlinear dynamical stochastic models capturing the observational phenomena. Open problems and future work will also be discussed.

There will be no final exam and homework. The enrolled students will participate actively in some lectures being mentored by Professor Majda and his two postdocs Nan Chen and Sulian Thual.

**Prerequisites:** undergraduate courses in ODEs and PDEs.
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El Niño is a climate pattern that includes the interactions between
1. atmosphere, 2. ocean, 3. sea surface temperature (SST).

El Niño & La Niña are the opposite phases of El Niño-Southern Oscillation (ENSO).
Remarkable Observational Phenomena of the ENSO

El Niño Southern Oscillation

- Warm phase: El Niño
- Cold phase: La Niña

Two types of super El Niño:
2) 2014-2016


Therefore, ENSO is more than a simple regular oscillator!
Remarkable Observational Phenomena of the ENSO

El Niño Southern Oscillation

▶ Warm phase: El Niño
▶ Cold phase: La Niña

Remarkable Observational Phenomena of the ENSO

El Niño Southern Oscillation

- Warm phase: El Niño
- Cold phase: La Niña

Two types of super El Niño:
1) 1982-1983 and 1997-1998,
2) 2014-2016.
Remarkable Observational Phenomena of the ENSO

El Niño Southern Oscillation

- Warm phase: El Niño
- Cold phase: La Niña


Remarkable Observational Phenomena of the ENSO

El Niño Southern Oscillation

- Warm phase: El Niño
- Cold phase: La Niña


Therefore, ENSO is more than a simple regular oscillator!
Remarkable Observational Phenomena of the ENSO: Global Warming and ENSO

Nino 3.4 Index

Years

Global mean surface temperature anomalies (°C)

Years
Remarkable Observational Phenomena of the ENSO: Global Warming and ENSO

**Nino 3.4 Index**

**Global mean surface temperature anomalies (°C)**

Global warming hiatus?

2001–2013
Remarkable Observational Phenomena of the ENSO: Global Warming and ENSO
Remarkable Observational Phenomena of the ENSO: Global Warming and ENSO

Nino 3.4 Index

Global mean surface temperature anomalies (°C)

Global warming hiatus?
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Nino 3.4 Index

Global mean surface temperature anomalies (°C)

Global warming hiatus?

Remarkable Observational Phenomena of the ENSO: Global Warming and ENSO

![Graph showing Nino 3.4 Index and Global mean surface temperature anomalies from 1940 to 2020.](image)

- **Nino 3.4 Index**
  - The graph displays the fluctuations in the Nino 3.4 index over the years 1940 to 2020.
  - The index values range from -3 to 3, with red and blue colors indicating positive and negative anomalies, respectively.

- **Global mean surface temperature anomalies (°C)**
  - The graph shows the global mean surface temperature anomalies from 2001 to 2013 and from 2014 to 2016.
  - The anomalies range from -0.4 to 1.0, with years indicated on the right side of the graph.
Remarkable Observational Phenomena of the ENSO: Two Types of Super El Niño


2014-2016: Delayed Super El Niño
Remarkable Observational Phenomena of the ENSO: El Niño Diversity

CP El Niño has been frequently observed since 1990.
Global Impact of ENSO

The anomalous climate patterns in the equatorial Pacific affect global climate through teleconnections, which are atmospheric interactions between widely separated regions.
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The anomalous climate patterns in the equatorial Pacific affect global climate through teleconnections, which are atmospheric interactions between widely separated regions.
Challenge and Goal of This Course

ENSO is an exciting but still not well-known subject.

Developing simplified mathematical models to

- explain the underlying mechanisms of ENSO,
- capture key observational features,
- provide suggestions for improving operational models.

The reanalysis data during the last 34 years will be used for observational surrogates.
Unrealistic behaviors in operational models ...

- unrealistic CP El Niño with locations far west in GFDL Climate Model (CM 2.1)
- obvious gap in power spectrum between model and observations
- only 10% models in CMIP3 and CMIP5 having the correct sign of Corr(N3, N4)
- many other problems with durations, amplitudes, locations etc...

CM2.1 simulation

ENSO amplitude & period
This course: A new modeling framework for ENSO

The new modeling framework contains simple ideas that combine

▶ linear and nonlinear dynamics,
▶ techniques from basic PDEs (e.g., the characteristic method),
▶ simple numerics, and
▶ novel stochastic tools (e.g., OU process + Markov jump process).

The modeling framework is based on

physical reasoning + rigorous mathematical justification

The mathematical part will be self-contained.

The course also aims at explaining how to deal with observational data.
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Suggested Reading:

Lecture 1, 2 and 3:
Handout notes by the instructors.

Lecture 4:

Lecture 5:

Lecture 6:

Lecture 7 and 8:

Lecture 9:

Lecture 10 and 11:
Sulian Thual, Andrew J. Majda and Nan Chen, Seasonal synchronization of a simple stochastic dynamical model capturing El Nino diversity, Submitted to *J. Climate* with minor revision, 2017

Lecture 12:
Sulian Thual, Andrew J. Majda and Nan Chen, Mechanisms of the 2014-2016 delayed super El Nino captured by simple dynamical models, Submitted to *Climate Dynamics*, 2017
Additional reading materials besides the class notes for Lecture 1, 2 and 3:


Thank you