Computer-Aided Design of MEMS

Eigenvalues, Energy Losses, and Dick Tracy Watches

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Outline

- MEMS (Micro-Electro-Mechanical Systems) basics and RF (Radio Frequency) MEMS
- Disk resonators and perfectly matched layers
- Beam resonators and thermoelastic damping
- Model reduction, mode tracking, and optimization
- Conclusions
What are MEMS?

MEMS Basics
- What are MEMS?
  - MEMS Basics
  - Fabrication outline
  - Fabrication result
  - Fabrication characteristics
  - RF MEMS
  - Micromechanical filters
  - Damping and Q
  - Sources of damping

Anchor loss
Thermoelastic damping
Filter design
Conclusions
MEMS Basics

- Micro-electro-mechanical systems
  - Chemical, fluid, thermal, optical (MECFTOMS?)

- Applications:
  - Sensors (inertial, chemical, pressure)
  - Ink jet printers, biolab chips
  - RF devices: cell phones, inventory tags, pico radio
  - “Smart dust”

- Use integrated circuit (IC) fabrication technology
- Large surface area / volume ratio
- Still mostly classical (vs. nanosystems)
Fabrication outline

1. Si wafer
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2. Deposit 2 microns SiO2
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2. Deposit 2 microns SiO2
3. Pattern and etch SiO2 layer
Fabrication outline

1. Si wafer
2. Deposit 2 microns SiO2
3. Pattern and etch SiO2 layer
4. Deposit 2 microns polycrystalline Si
1. Si wafer
2. Deposit 2 microns SiO2
3. Pattern and etch SiO2 layer
4. Deposit 2 microns polycrystalline Si
5. Pattern and etch Si layer
Fabrication outline

1. Si wafer
2. Deposit 2 microns SiO2
3. Pattern and etch SiO2 layer
4. Deposit 2 microns polycrystalline Si
5. Pattern and etch Si layer
6. Release etch remaining SiO2
Fabrication result

(C. Nguyen, iMEMS 01)
Fabrication characteristics

- Characteristic dimensions: microns
- Geometry is “2.5” dimensional
- Relatively loose fabrication tolerances
- Difficult to characterize
Microguitars from Cornell University (1997 and 2003)

- MHz-GHz mechanical resonators
- Impact: smaller, lower-power cell phones
  - Replace quartz freq references, filter elements
  - Integrate into CMOS stack
- Other uses:
  - Sensing elements (e.g. chemical sensors)
  - Really high-pitch guitars
Micromechanical filters

- Mechanical high-frequency (high MHz-GHz) filter
  - Your cell phone is mechanical!

- Advantage over quartz surface acoustic wave filters
  - Integrated into chip
  - Low power

Success $\implies$ "Calling Dick Tracy!"
Damping and $Q$

- Want to minimize damping
  - Electronic filters have too much
  - Understanding of damping in MEMS resonators is lacking
- Engineers want one number: $Q$
  - Non-dimensionalized damping in a one-variable system:
    \[
    \frac{d^2u}{dt^2} + Q^{-1}\frac{du}{dt} + u = F(t)
    \]
  - For a resonant mode with frequency $\omega \in \mathbb{C}$:
    \[
    Q := \frac{\Re(\omega)}{2\Im(\omega)} = \frac{\text{Stored energy}}{\text{Energy loss per radian}}
    \]
- Goal: Make $Q$ big
Sources of damping

- Fluid damping
  - Air is a viscous fluid \((\text{Re} \ll 1)\)
  - Can operate in a vacuum
  - Shown not to dominate in many RF designs

- Anchor loss
  - Elastic waves radiate from structure

- Thermoelastic damping
  - Volume changes induce temperature change
  - Diffusion of heat leads to mechanical loss

- Material losses (catch-all)
  - Low intrinsic losses in silicon, diamond, germanium, etc.
  - Terrible material losses in metals
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Disk resonator

- SiGe disk resonators built by E. Quévy
- Axisymmetric model with bicubic mesh, about 10K nodal points
Substrate model

Goal: Understand energy loss in this resonator
- Dominant loss is elastic radiation from anchor.
- Disk resonator is much smaller than substrate
- Very little energy leaving the post is reflected back
  - Substrate is semi-infinite from disk’s perspective
- Possible semi-infinite models
  - Matched asymptotic modes
  - Dirichlet-to-Neumann maps
  - Boundary dampers
  - Perfectly matched layers
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Perfectly matched layers

- Apply a complex coordinate transformation
- Generates a non-physical absorbing layer
- No impedance mismatch between the computational domain and the absorbing layer
- Idea works with general linear wave equations
  - First applied to Maxwell’s equations (Berengér 95)
  - Similar idea introduced earlier in quantum mechanics (*exterior complex scaling*, Simon 79)
Scalar wave example

Outgoing wave $\exp(-iz)$

Incoming wave $\exp(iz)$

Transformed coordinate $z = x + iy$
Scalar wave example

Outgoing wave $\exp(-iz)$

Incoming wave $\exp(iz)$

Transformed coordinate $z = x + iy$
Scalar wave example

Outgoing wave exp(−iz)

Incoming wave exp(iz)

Transformed coordinate z = x + iy
Scalar wave example

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Incoming wave exp(iz)

Transformed coordinate \( z = x + iy \)
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Scalar wave example

Outgoing wave $\exp(-iz)$

Incoming wave $\exp(iz)$

Transformed coordinate $z = x + iy$

Clamp solution at transformed end to isolate outgoing wave.
PML weak form

Weak form of time-harmonic elasticity equations:

\[-\omega^2 \int_{\Omega} \rho w \cdot u \, d\Omega + \int_{\Omega} \epsilon(w) : C : \epsilon(u) \, d\Omega = \int_{\Gamma} w \cdot t \, d\Gamma\]

Weak form of time-harmonic PML equation:

\[-\omega^2 \int_{\Omega} \rho w \cdot u \, Jd\Omega + \int_{\Omega} \tilde{\epsilon}(w) : C : \tilde{\epsilon}(u) \, Jd\Omega = \int_{\Gamma} w \cdot \tilde{t} \, Jd\Gamma\]

Bubnov-Galerkin finite element discretization leads to

\[-\omega^2 M u + K u = F\]

But in PML case, \(M\) and \(K\) are complex symmetric.
Finite elements

- Isoparametric elements already use mapped integration
- View PML as an added coordinate transformation
  - Requires little modification to existing elements
  - Just transform derivatives and Jacobian determinant
Eigenstructure

- Complex symmetry implies row and column eigenvectors are (non-conjugated) transposes.
- Can therefore achieve second-order accuracy with a modified Rayleigh quotient:

\[ \theta(v) = \frac{(v^T K v)}{(v^T M v)} \]

- It is possible to have \( v^T M v \approx 0 \)
  - Propagating modes (continuous spectrum)
  - Not the modes of interest for resonators
Perturbation analysis

Know first-order perturbation behavior of eigenvalues:

$$(\omega + \delta \omega)^2 = \frac{v^T (K + \delta K)v}{v^T (M + \delta M)v}$$

Useful for sensitivity analysis.
Model reduction

Would like a reduced model which
- Preserves second-order accuracy for converged eigs
- Keeps at least Arnoldi’s accuracy otherwise
- Is physically meaningful

Idea:
- Build an Arnoldi basis $V$
- Double the size: $W = \text{orth}(\Re(V), \Im(V))$
- Use $W$ as a projection basis
- Resulting system is still a Galerkin approximation with real shape functions for the continuum PML equations
- Compute complex frequencies by shift-and-invert Arnoldi with an analytically determined shift
- Surprising variation – experimentally observed – in $Q$ as film thickness changes!
Effect of varying film thickness

- Sudden dip in $Q$ comes from an interaction between a (mostly) bending mode and a (mostly) radial mode
- Non-normal interaction between the modes
Truth in advertising

Data from a set of 30μm radius disks.
Thermoelastic damping (TED)

\[ u \text{ is displacement and } T = T_0 + \theta \text{ is temperature} \]

\[
\sigma = C\varepsilon - \beta \theta I \\
\rho u_{tt} = \nabla \cdot \sigma \\
\rho c_v \theta_t = \nabla \cdot (\kappa \nabla \theta) - \beta T_0 \text{tr}(\varepsilon_t)
\]

- Volumetric strain rate drives energy transfer from mechanical to thermal domain
  - Irreversible diffusion \(\implies\) mechanical damping
  - Not often an important factor at the macro scale
  - Recognized source of damping in microresonators

- Zener: semi-analytical approximation for TED in beams
- We consider the fully coupled system
Nondimensionalization

\[
\sigma = \hat{C}e - \zeta \theta 1
\]

\[
u_{tt} = \nabla \cdot \sigma
\]

\[
\theta_t = \eta \nabla^2 \theta - \text{tr}(\epsilon_t)
\]

\[
\zeta := \left( \frac{\beta}{\rho c} \right)^2 \frac{T_0}{c_v} \quad \text{and} \quad \eta := \frac{\kappa}{\rho c_v c L}
\]

Length \quad \sim \quad L

Time \quad \sim \quad L/c, \text{ where } c = \sqrt{E/\rho}

Temperature \quad \sim \quad T_0 \frac{\beta}{\rho c_v}
Scaling analysis

\[ \sigma = \hat{C} \epsilon - \zeta \theta I \]
\[ u_{tt} = \nabla \cdot \sigma \]
\[ \theta_t = \eta \nabla^2 \theta - \text{tr}(\epsilon_t) \]

\[ \zeta := \left( \frac{\beta}{\rho c} \right)^2 \frac{T_0}{c_v} \quad \text{and} \quad \eta := \frac{k}{\rho c_v c_L} \]

- Micron-scale poly-Si devices: \( \zeta \) and \( \eta \) are \( \sim 10^{-4} \).
- Small \( \eta \) leads to thermal boundary layers
- Linearize about \( \zeta = 0 \)
Discrete mode equations

\[ \sigma = \hat{C}\epsilon - \xi \theta_1 \]
\[ u_{tt} = \nabla \cdot \sigma \]
\[ \theta_t = \eta \nabla^2 \theta - \text{tr}(\epsilon_t) \]

\[ \sigma = \hat{C}\epsilon - \xi \theta_1 \]
\[ -\omega^2 u = \nabla \cdot \sigma \]
\[ i\omega \theta = \eta \nabla^2 \theta - i\omega \text{tr}(\epsilon) \]

\[ -\omega^2 M_{uu}u + K_{uu}u + K_{ut}\theta = 0 \]
\[ i\omega D_{tt}\theta + K_{tt}\theta + i\omega D_{tu}u = 0 \]
Perturbation computation

\[-\omega^2 M_{uu} u + K_{uu} u + K_{ut} \theta = 0\]
\[i \omega D_{tt} \theta + K_{t\theta \theta} + i \omega D_{tu} u = 0\]

Approximate \( \omega \) by perturbation about \( K_{ut} = 0 \):

\[-\omega_0^2 M_{uu} u_0 + K_{uu} u_0 = 0\]
\[i \omega_0 D_{tt} \theta_0 + K_{t\theta \theta} + i \omega_0 D_{tu} u_0 = 0\]

Choose \( v : v^T u_0 \neq 0 \) and compute

\[
\begin{bmatrix}
-\omega_0^2 M_{uu} + K_{uu} & -2 \omega_0 M_{uu} u_0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta u \\
\delta \omega
\end{bmatrix} =
\begin{bmatrix}
-K_{ut} \theta_0 \\
0
\end{bmatrix}
\]
Comparison to Zener’s model

- Comparison of fully coupled simulation to Zener approximation over a range of frequencies
- Real and imaginary parts after first-order correction agree to about three digits with Arnoldi
Checkerboard resonator

- Array of loosely coupled resonators
- Anchored at outside corners
- Excited at northwest corner
- Sensed at southeast corner
- Surfaces move only a few nanometers
Checkerboard simulation
Checkerboard measurement

S. Bhave, MEMS 05
Transfer function optimization

- Choose geometry to make a good bandpass filter
- What is a “good bandpass filter?”
  - $|H(\omega)|$ is big on $[\omega_l, \omega_r]\$
  - $|H(\omega)|$ is tiny outside this interval
- How do we optimize?
  - Overton’s gradient sampling method
  - Use Byers-Boyd-Balikrishnan algorithm for distance to instability to minimize $|H(\omega)|$ on $[\omega_l, \omega_r]$
  - Small Hamiltonian eigenproblem (with ROM)
Contributions

- Mathematical
  - Reformulation of PML technology
  - Perturbation solution for thermoelastic damping

- Computer science
  - HiQLab, SUGAR, and FEAPMEX

- Engineering physics
  - Effects of mode interference in damping
  - Relative importance of anchor loss and TED
The other talk

- CLAPACK
- Finding roots of polynomials
- Continuation of invariant subspaces for sparse problems
- Computer network tomography
- OceanStore and distributed system security
- Pontificating about floating point arithmetic
Conclusions

- RF MEMS are a great source of problems
  - Interesting applications
  - Interesting physics (and not altogether understood)
  - Interesting numerical mathematics

http://www.cs.berkeley.edu/~dbindel/feapmex.html
http://www.cs.berkeley.edu/~dbindel/hiqlab
http://bsac.berkeley.edu/cadtools/sugar/sugar/
Role of simulation

HiQLab: Modeling RF MEMS
- Explore fundamental device physics
  ♦ Particularly details of damping
- Detailed finite element modeling
- Reduced models eventually to go into SUGAR

SUGAR: “Be SPICE to the MEMS world”
- Fast enough for early design stages
- Simple enough to attract users
- Support design, analysis, optimization, synthesis
- Verify models by comparison to measurement
Shear ring resonator

- Ring is driven in a shearing motion
- Can couple ring to other resonators
- How do we track the desired mode?
Mode tracking

Find a continuous solution to

\[
\left( K(s) - \omega(s)^2 M(s) \right) u(s) = 0.
\]

- \( K \) and \( M \) are symmetric and \( M > 0 \)
- Eigenvectors are \( M \)-orthogonal
- Perturbation theory gives good shifts
- Look if \( u(s + h) \) and \( u(s) \) are on the same path by looking at \( u(s + h)^T M(s + h) u(s) \)
- Many more subtleties in the nonsymmetric case
  - Focus of the \textit{CIS algorithm}
Mode tracking in a shear resonator

Frequency (Hz)

Half width of annulus

Finite element

Analytic result

° Finite element

° Analytic result
Thermoelastic boundary layer

- One-dimensional test problem (longitudinal mode in a bar)
- Fixed temperature and displacement at left
- Free at right