Table 1: Summary of the regression models for the PDO (L_1) and NPGO (L_2) modes from equations (1) and (2) in the main text. $\{\mathbf{A}_i, \mathbf{B}_i, \mathbf{C}_i\}$: Regression coefficients. σ_i: Standard deviation of the residual. κ_i: Condition number of the design matrix. $\{d(\epsilon_i), d(\Delta\epsilon_i)\}$: Durbin-Watson statistics for the residual and residual increments for 1 m lag. $t_c(\epsilon_i)/t_c(L_i)$: noise decorrelation time relative to modal decorrelation time. ✓ (X) indicates external factors/interactions which were (were not) part of a given model.

Model	1	2	3	4	5
Ext. factors					
P_1, P_2	X	X	✓	✓	✓
L_3	X	✓	X	X	X
$I_1, \ldots, I_4$	X	X	✓	✓	✓
Mode L_1					
$\mathbf{A}_1(L_1)$	0.9964	0.9972	X	0.6233	−0.0067
$\mathbf{A}_1(L_2)$	0.0343	0.0330	X	−0.0624	−0.2148
$\mathbf{B}_1(L_3)$	X	−0.0340	X	X	X
$\mathbf{B}_1(P_1, I_1)$	X	X	0.4070	0.1732	0.4606
$\mathbf{B}_1(P_2, I_2)$	X	X	0.4762	0.1954	0.5241
$\mathbf{B}_1(P_1, I_3)$	X	X	0.2709	0.0907	0.2508
$\mathbf{B}_1(P_2, I_3)$	X	X	−0.1778	−0.0180	−0.0598
$\mathbf{C}_1(L_1, L_1, L_1)$	X	X	X	X	−0.0032
$\mathbf{C}_1(L_1, L_2, L_2)$	X	X	X	X	0.0046
σ_1	0.0735	0.0658	0.1582	0.0696	0.0800
κ_1	1.1779	1.2138	2.1941	40.184	7.008
$d(\epsilon_1)$	0.5862	0.7428	0.5886	0.6284	0.6810
$d(\Delta\epsilon_1)$	2.0342	2.0190	1.3244	2.1196	1.9682
$t_c(\epsilon_1)/t_c(L_1)$	0.2307	0.1954	0.3388	0.1967	0.1621
Mode L_2					
$\mathbf{A}_2(L_1)$	−0.0282	−0.0278	X	−0.0303	−0.1787
$\mathbf{A}_2(L_2)$	0.9916	0.9086	X	0.9451	−0.0107
$\mathbf{B}_2(L_3)$	X	−0.0197	X	X	X
$\mathbf{B}_2(P_1, I_1)$	X	X	0.1172	0.0001	0.2144
$\mathbf{B}_2(P_2, I_1)$	X	X	−0.2610	−0.0409	−0.2397
$\mathbf{B}_2(P_2, I_2)$	X	X	0.1604	0.0196	0.2418
$\mathbf{B}_2(P_2, I_3)$	X	X	0.4128	0.0540	0.3653
$\mathbf{B}_2(P_1, I_4)$	X	X	−0.4850	0.0128	−0.4414
$\mathbf{B}_2(P_2, I_4)$	X	X	0.0118	0.0275	−0.0358
$\mathbf{C}_2(L_1, L_1, L_2)$	X	X	X	X	0.0141
$\mathbf{C}_2(L_2, L_2, L_2)$	X	X	X	X	0.0048
σ_2	0.1330	0.1316	0.1590	0.1167	0.1573
κ_2	1.1779	1.2138	2.3206	23.612	46.400
$d(\epsilon_2)$	0.8099	0.8277	0.5262	0.7512	0.5213
$d(\Delta\epsilon_2)$	1.9709	1.9711	2.0206	2.2984	2.0615
$t_c(\epsilon_2)/t_c(L_2)$	0.2270	0.2270	0.2220	0.2257	0.2159

Table 2: Properties of mode I_1 in the regression models for the intermittent modes $(I_1, \ldots, I_4)$ from equation (3) in the main text. $\{\mathbf{A}_i, \mathbf{B}_i, \mathbf{C}_i\}$: Regression coefficients. σ_i: Standard deviation of the residual. κ_i: Condition number of the design matrix. $\{d(\epsilon_i), d(\Delta\epsilon_i)\}$: Durbin-Watson statistics for the residual and residual increments for 1 m lag. $t_c(\epsilon_i)/t_c(I_i)$: noise decorrelation time relative to modal decorrelation time. ✓ (X) indicates external factors/interactions which were (were not) part of a given model.

Model	1	2	3	4
Ext. factors				
P_1, P_2	X	✓	✓	✓
L_1, L_2	X	✓	✓	✓
$\mathbf{A}_1(I_1)$	0.8787	X	0.2275	-0.0278
$\mathbf{A}_1(I_2)$	-0.4118	X	-0.0618	0.0298
$\mathbf{A}_1(I_3)$	0.0348	X	-0.2505	-0.3228
$\mathbf{A}_1(I_4)$	0.2241	X	0.8530	-0.1051
$\mathbf{B}_1(P_1, L_1)$	X	0.7031	0.0420	-0.4160
$\mathbf{B}_1(P_2, L_1)$	X	-0.3625	-0.1865	-0.5110
$\mathbf{B}_1(P_2, L_2)$	X	-0.6142	0.0091	0.9017
$\mathbf{C}_1(I_1, I_1, I_1)$	X	X	X	0.0155
$\mathbf{C}_1(I_1, I_2, I_2)$	X	X	X	-0.0325
$\mathbf{C}_1(I_2, I_2, I_2)$	X	X	X	0.0044
$\mathbf{C}_1(I_1, I_3, I_3)$	X	X	X	0.0172
$\mathbf{C}_1(I_1, I_4, I_4)$	X	X	X	-0.0147
$\mathbf{C}_1(I_4, I_4, I_4)$	X	X	X	-0.0050
σ_1	0.1077	0.1973	0.1056	0.2140
κ_1	1.1180	1.1629	22.758	21.880
$d(\epsilon_1)$	0.8062	0.4021	0.8444	0.5977
$d(\Delta\epsilon_1)$	1.8797	1.3187	1.8916	1.4349
$t_c(\epsilon_1)/t_c(I_1)$	0.1895	0.1892	0.3009	0.3181

Table 3: Same as Table 2, but for mode I_2

Model	1	2	3	4
Ext. factors				
P_1, P_2	X	✓	✓	✓
L_1, L_2	X	✓	✓	✓
$\mathbf{A}_2(I_1)$	0.4168	X	0.3446	-0.0975
$\mathbf{A}_2(I_2)$	0.8603	X	0.7029	0.0058
$\mathbf{A}_2(I_3)$	0.2701	X	0.4159	0.2879
$\mathbf{A}_2(I_4)$	-0.0800	X	-0.1207	-0.2679
$\mathbf{B}_2(P_1, L_1)$	X	0.6157	0.0196	0.6017
$\mathbf{B}_2(P_2, L_1)$	X	0.7006	0.2062	0.8294
$\mathbf{B}_2(P_2, L_2)$	X	0.3065	-0.1131	-0.0323
$\mathbf{C}_2(I_1, I_1, I_1)$	X	X	X	-0.0010
$\mathbf{C}_2(I_1, I_1, I_2)$	X	X	X	-0.0057
$\mathbf{C}_2(I_2, I_2, I_2)$	X	X	X	-0.0007
σ_2	0.1109	0.2484	0.1059	0.1355
κ_2	1.1180	1.1629	22.758	43.016
$d(\epsilon_2)$	1.0876	0.4758	1.1286	0.8238
$d(\Delta\epsilon_2)$	1.9688	1.1089	1.9597	1.7343
$t_c(\epsilon_2)/t_c(I_2)$	0.4533	0.4458	0.5006	0.4217

Table 4: Same as Table 2, but for mode I_3

Model	1	2	3	4
Ext. factors				
P_1, P_2	X	✓	✓	✓
L_1, L_2	X	✓	✓	✓
$\mathbf{A}_3(I_1)$	-0.0493	X	0.0161	0.0909
$\mathbf{A}_3(I_2)$	-0.2835	X	-0.5005	0.0034
$\mathbf{A}_3(I_3)$	0.8449	X	0.7992	-0.0295
$\mathbf{A}_3(I_4)$	-0.4260	X	-0.4912	0.0124
$\mathbf{B}_3(P_1, L_1)$	X	0.2527	0.0237	0.1945
$\mathbf{B}_3(P_2, L_1)$	X	-0.6040	0.1872	-0.6298
$\mathbf{B}_3(P_1, L_2)$	X	0.2921	-0.0811	0.2740
$\mathbf{B}_3(P_2, L_2)$	X	0.7107	0.1303	0.8137
$\mathbf{C}_3(I_2, I_2, I_2)$	X	X	X	-0.0016
$\mathbf{C}_3(I_1, I_1, I_3)$	X	X	X	0.0112
$\mathbf{C}_3(I_2, I_2, I_3)$	X	X	X	-0.0005
$\mathbf{C}_3(I_3, I_3, I_3)$	X	X	X	-0.0090
$\mathbf{C}_3(I_3, I_4, I_4)$	X	X	X	-0.0078
$\mathbf{C}_3(I_4, I_4, I_4)$	X	X	X	0.0027
σ_3	0.1345	0.2090	0.1302	0.2246
κ_3	1.1180	1.1709	22.846	21.913
$d(\epsilon_3)$	0.8541	0.5706	0.8448	0.5541
$d(\Delta\epsilon_3)$	1.7568	1.3992	1.8681	1.3739
$t_c(\epsilon_3)/t_c(I_3)$	0.1985	0.1820	0.1887	0.1954

Table 5: Same as Table 2, but for mode I_4

Model	1	2	3	4
Ext. factors				
P_1, P_2	X	✓	✓	✓
L_1, L_2	X	✓	✓	✓
$\mathbf{A}_4(I_1)$	-0.2194	X	0.6970	-0.0051
$\mathbf{A}_4(I_2)$	0.0537	X	0.1936	-0.0304
$\mathbf{A}_4(I_3)$	0.4351	X	-0.0378	-0.0840
$\mathbf{A}_4(I_4)$	0.8534	X	-0.0795	0.0160
$\mathbf{B}_4(P_1, L_1)$	X	0.2476	0.0636	0.2693
$\mathbf{B}_4(P_1, L_2)$	X	-0.8812	-0.2075	-0.9084
$\mathbf{B}_4(P_2, L_2)$	X	0.2408	0.2586	0.3034
$\mathbf{C}_4(I_3, I_3, I_3)$	X	X	X	-0.0009
$\mathbf{C}_4(I_1, I_1, I_4)$	X	X	X	-0.0093
$\mathbf{C}_4(I_3, I_3, I_4)$	X	X	X	0.0017
$\mathbf{C}_4(I_4, I_4, I_4)$	X	X	X	-0.0014
σ_4	0.1677	0.1981	0.1561	0.1869
κ_4	1.1180	1.1670	21.077	43.075
$d(\epsilon_4)$	0.6916	0.1659	0.8271	0.7822
$d(\Delta\epsilon_4)$	2.0263	1.6131	1.9311	1.6549
$t_c(\epsilon_4)/t_c(I_4)$	0.3029	0.2260	0.2538	0.1975

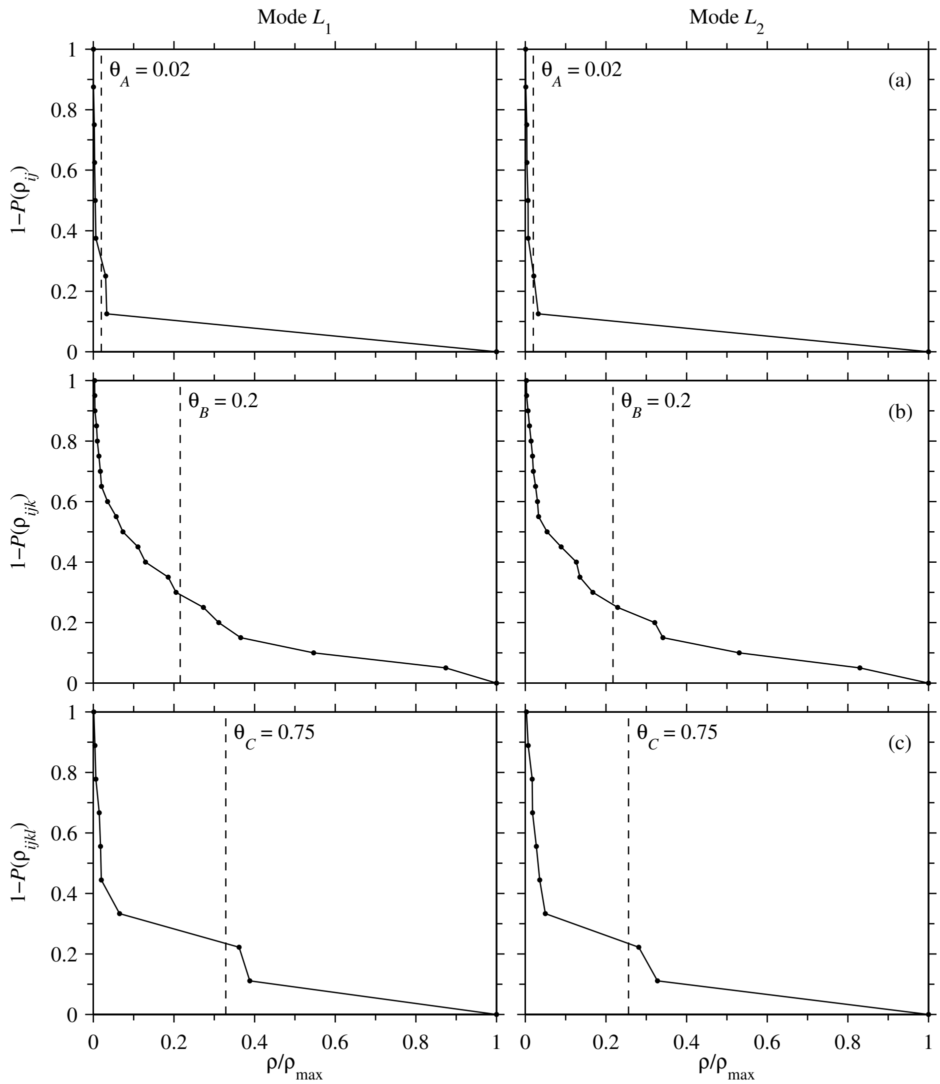

Figure 1: Cumulative distribution functions of the absolute values of the (a) double, $\rho_{ij}(t) = \langle v_i(\tau)L_j(\tau + t)\rangle$, (b) triple, $\rho_{ijk}(0, t) = \langle v_i(\tau)v_j(\tau)L_k(\tau + t)\rangle$, and (c) quadruple, $\rho_{ijkl}(0, 0, t) = \langle v_i(\tau)v_j(\tau)v_k(\tau)L_l(\tau + t)\rangle$, correlation coefficients evaluated for the low-frequency modes, $\{L_1, L_2\} = \{v_5, v_6\}$ at lead time $t = 1$ m. The predictor variables $v_i(\tau)$ are (a) modes $P_1, P_2, L_1, L_2, L_3, I_1, \ldots, I_4$, (b) double products of modes $P_1, P_2, I_1, \ldots, I_4$, (c) triple products of modes L_1, L_2, L_3. In each panel, the horizontal axis has been scaled by the maximum absolute value $\rho_{\max}$ of the correlation coefficients in the corresponding group. The dashed vertical lines indicate the θ values used for univariate thresholding. The admissible interactions lie to the right of those lines. Note that the coefficients in (c) involving mode L_3 were not used for regression modeling, but are included here for reference.

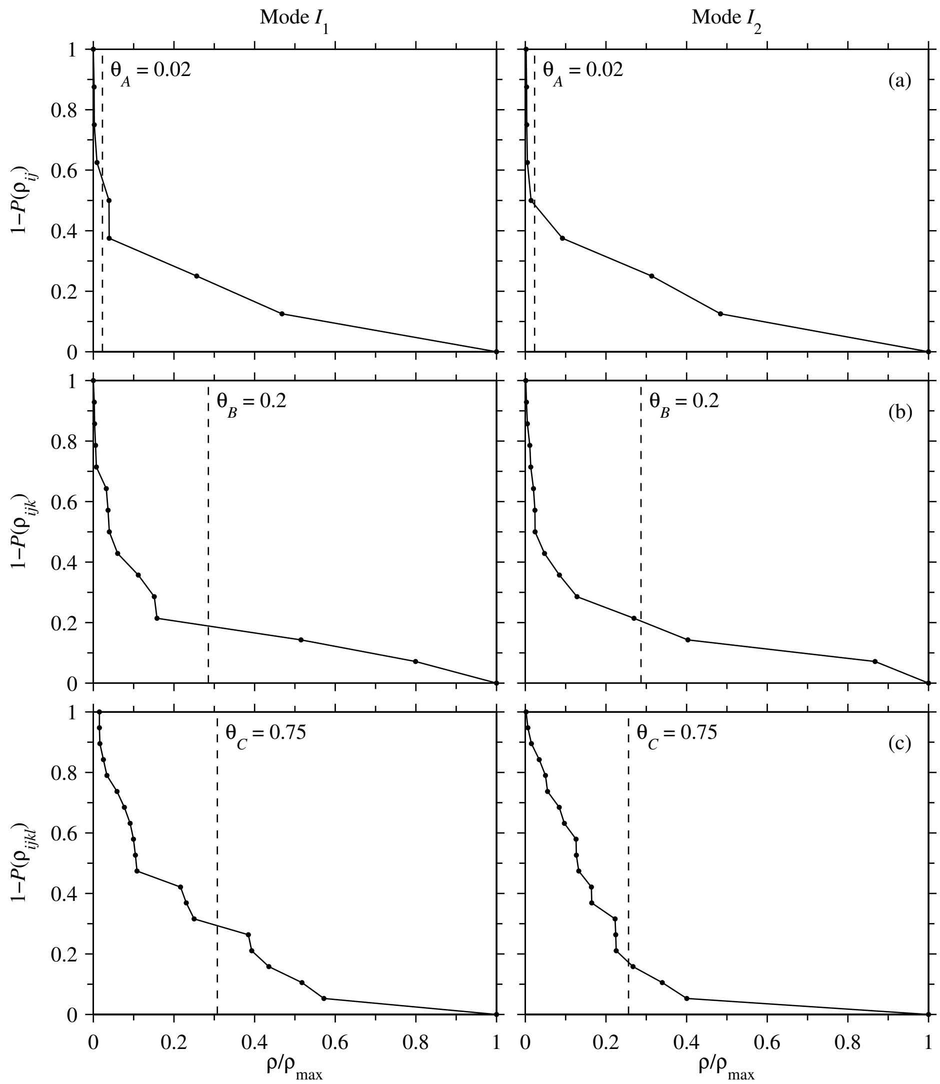

Figure 2: Cumulative distribution functions of the absolute values of the (a) double, $\rho_{ij}(t) = \langle v_i(\tau) I_j(\tau + t)\rangle$, (b) triple, $\rho_{ijk}(0,t) = \langle v_i(\tau) v_j(\tau) I_k(\tau + t)\rangle$, and (c) quadruple, $\rho_{ijkl}(0,0,t) = \langle v_i(\tau) v_j(\tau) v_k(\tau) I_l(\tau + t)\rangle$ evaluated for intermittent modes $\{I_1, I_2\} = \{v_{10}, v_{11}\}$ at lead time $t = 1$ m. The predictor variables $v_i(\tau)$ are (a) modes $P_1, P_2, L_1, L_2, L_3, I_1, \ldots, I_4$, (b) double products of modes P_1, P_2, L_1, L_2, L_3, (c) triple products of modes $I_1, \ldots, I_4$. In each panel, the horizontal axis has been scaled by the maximum absolute value $\rho_{\max}$ of the correlation coefficients in the corresponding group. The dashed vertical lines indicate the θ values used for univariate thresholding. The admissible interactions lie to the right of those lines.

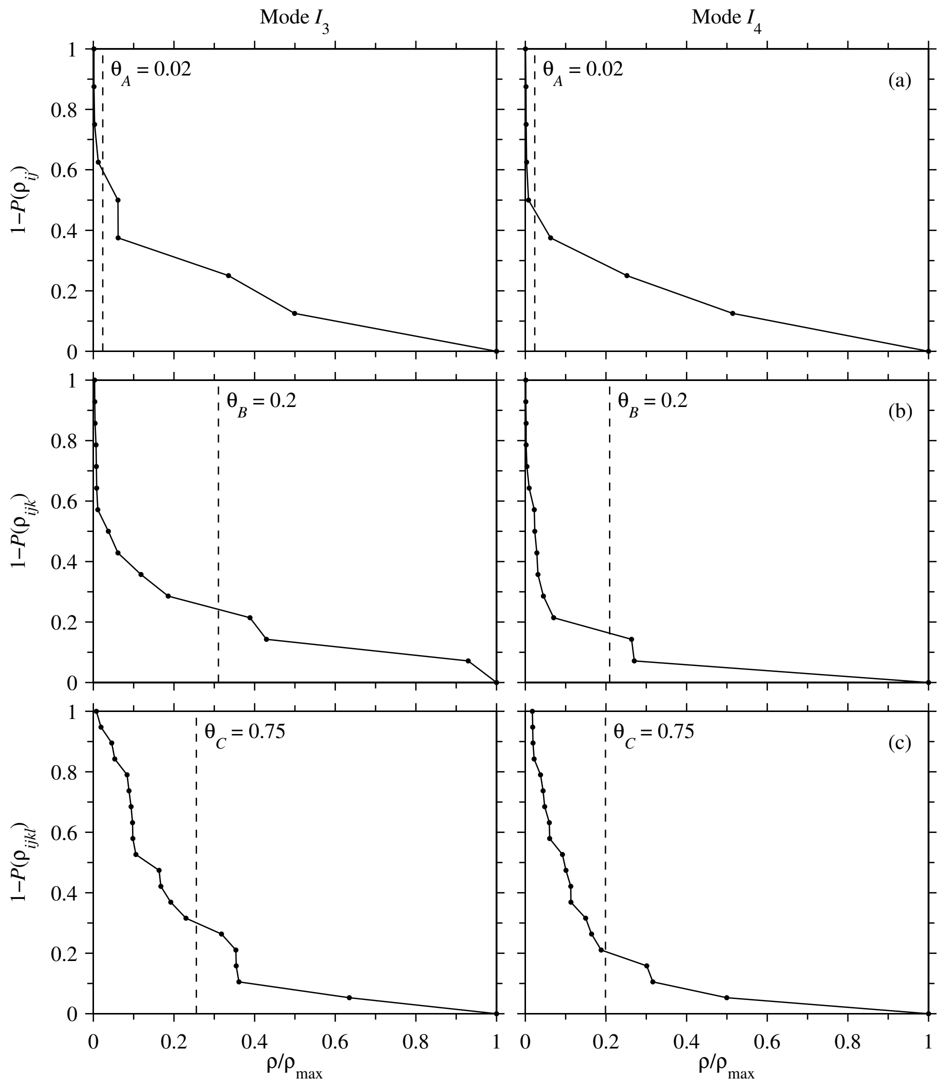

Figure 3: Same as Figure 2, but for intermittent modes $\{I_3, I_4\} = \{v_{12}, v_{13}\}$

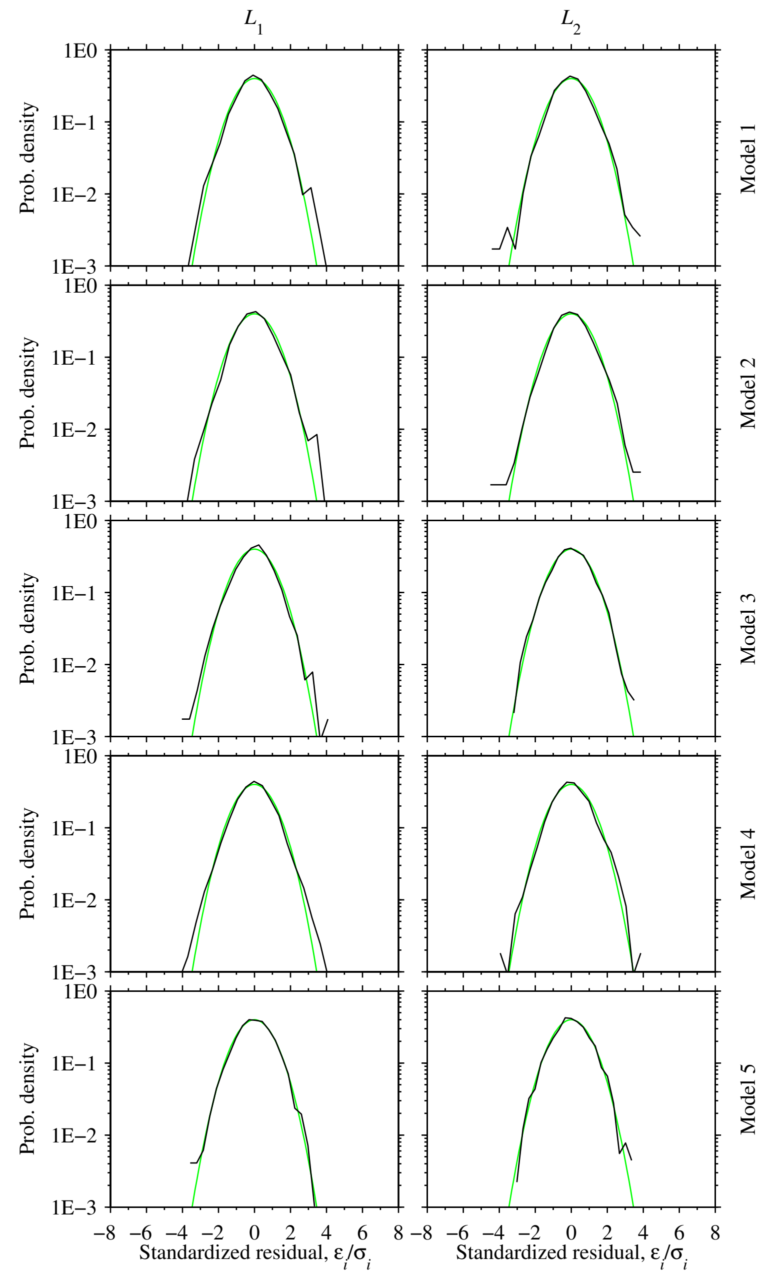

Figure 4: Residual histograms for models 1–5 in Table 1. The empirical histograms (black lines) were computed by binning the $n = 2687$ monthly samples $\epsilon_i(t)$ in the training time-series into $b = 20$ uniform bins in the range $[\min \epsilon_i(t), \max \epsilon_i(t)]$. Gaussian distributions with zero mean and unit variance are plotted in green lines for reference.

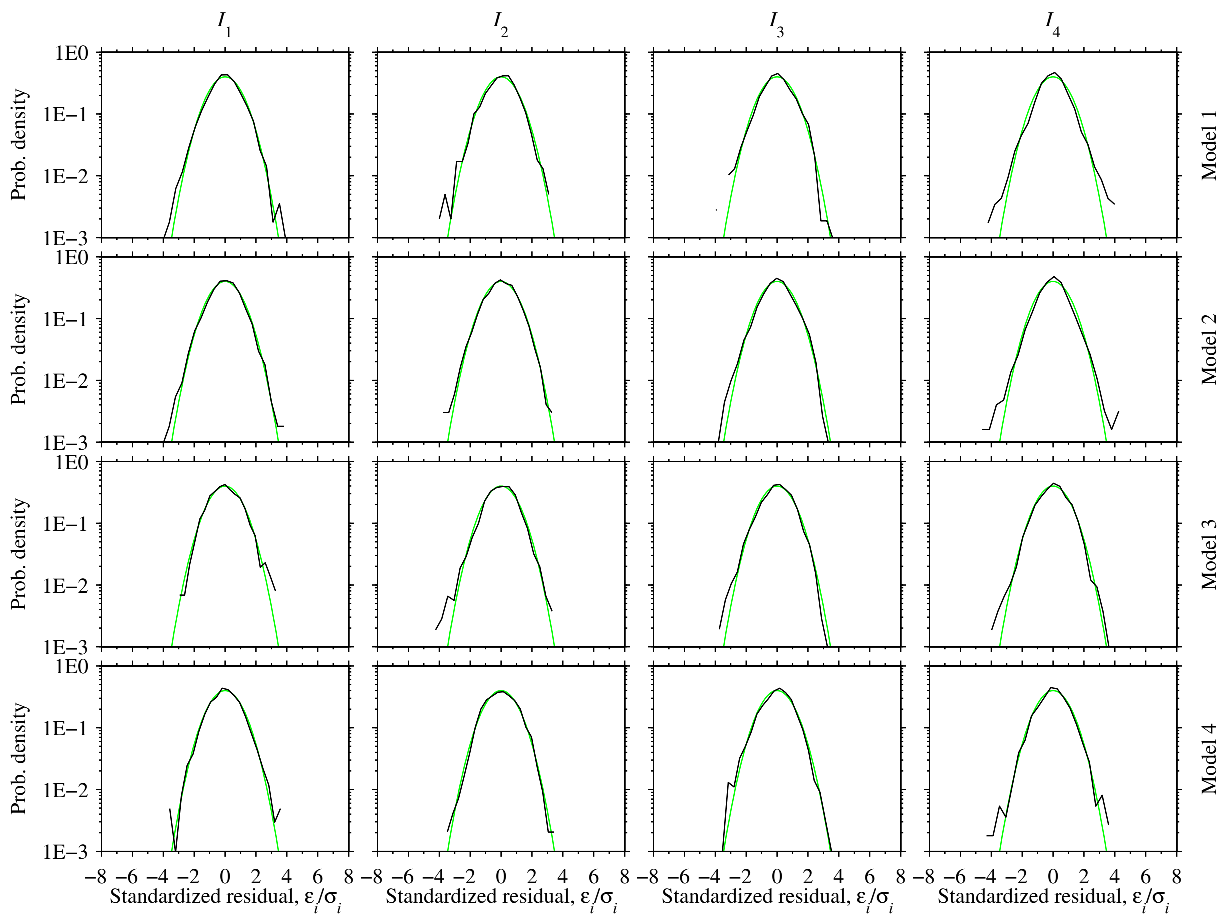

Figure 5: Same as Figure 4, but for models 1–4 in Tables 2–5

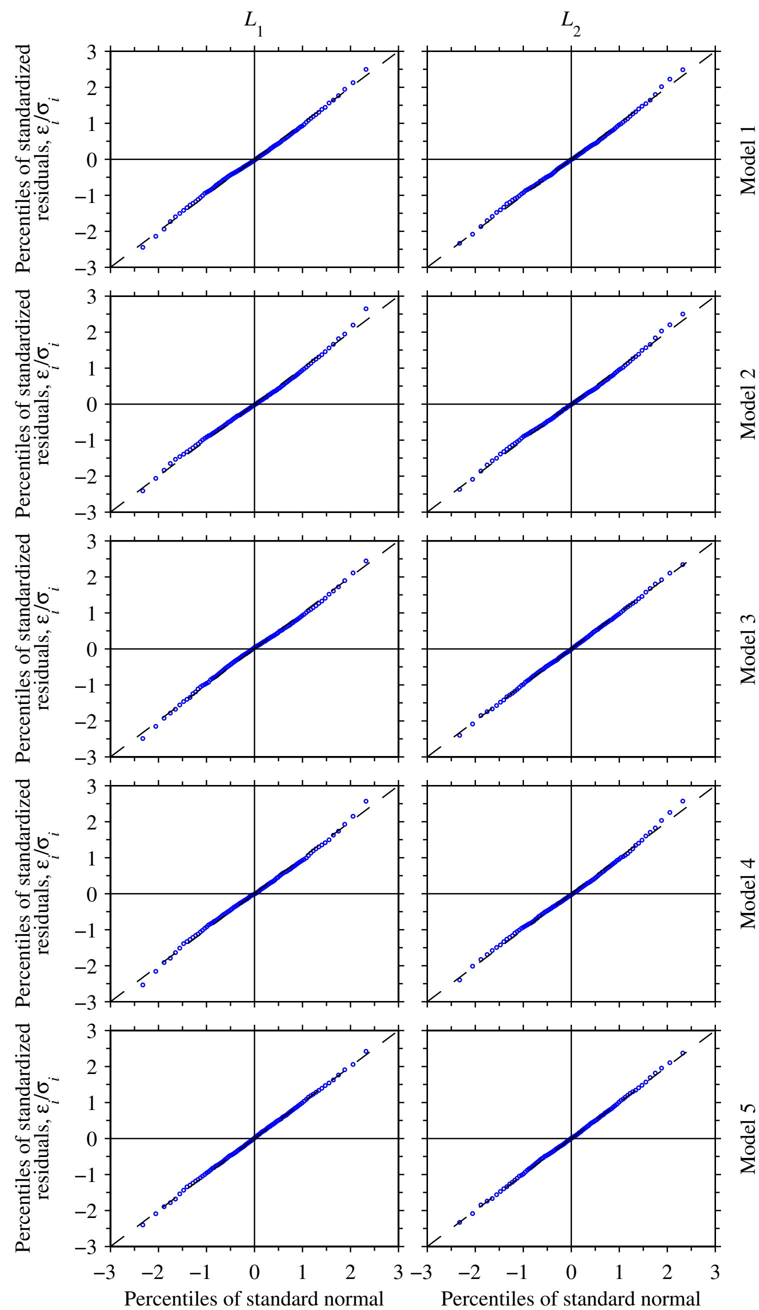

Figure 6: Quantile-quantile plots of the standardized residuals from models 1–5 in Table 1 versus standard normal variables

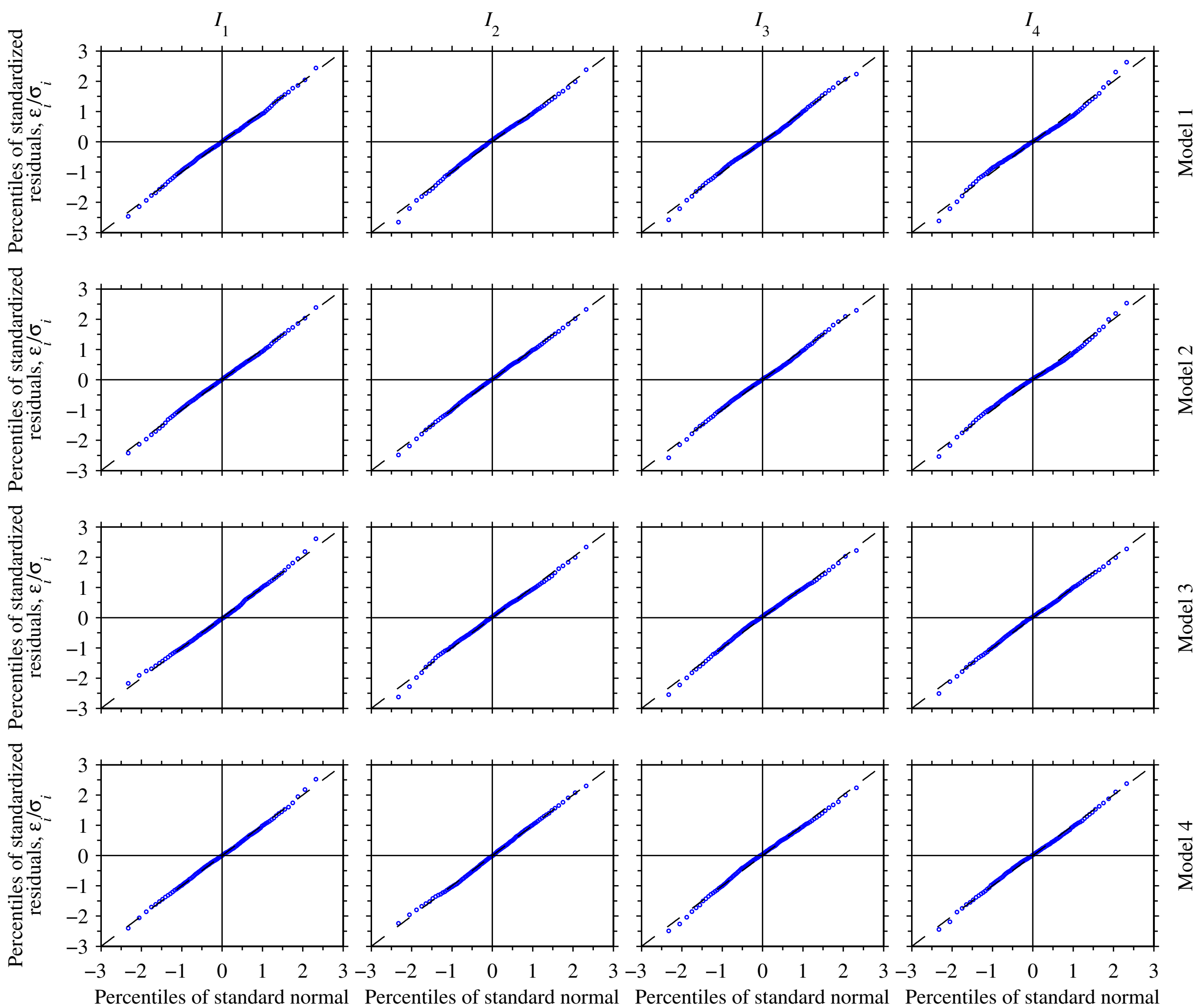

Figure 7: Quantile-quantile plots of the standardized residuals from models 1–4 in Tables 2–5 versus standard normal variables

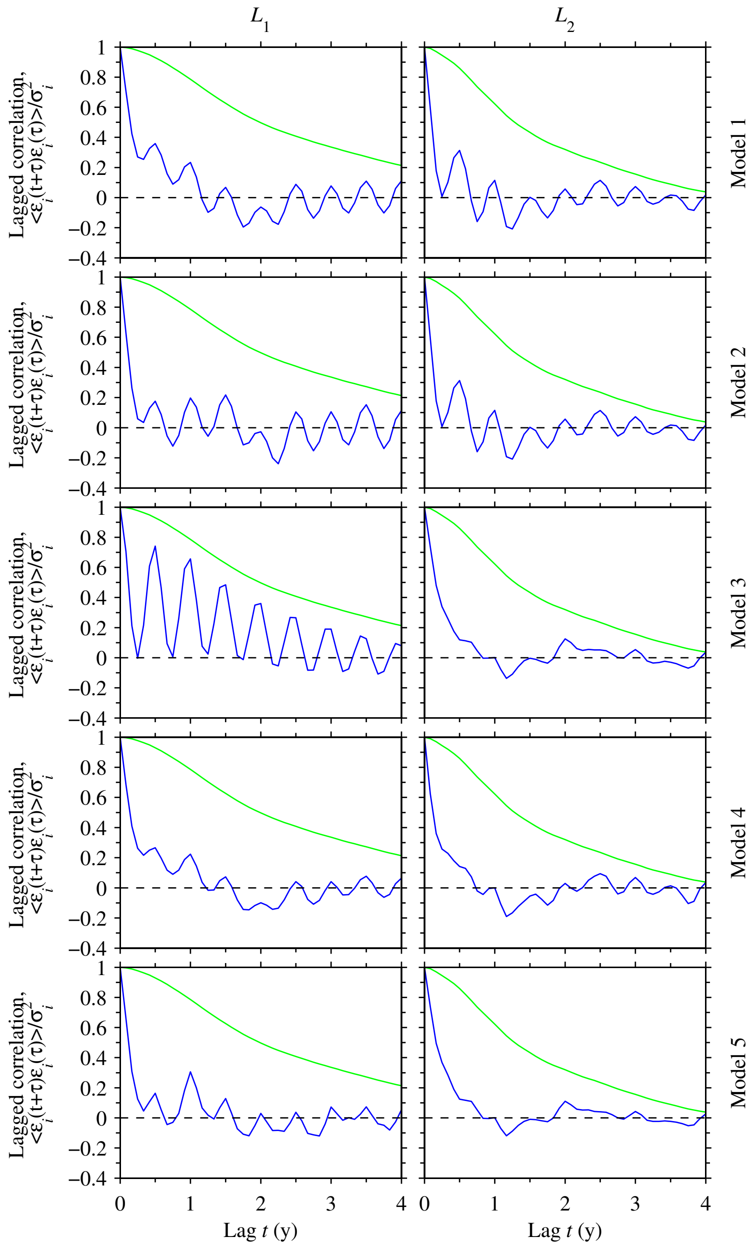

Figure 8: Lagged correlation coefficients of the residuals from models 1–5 in Table 1 (blue lines). Shown for reference are the lagged correlation coefficients of the corresponding response variables, L_1 and L_2 (green lines).

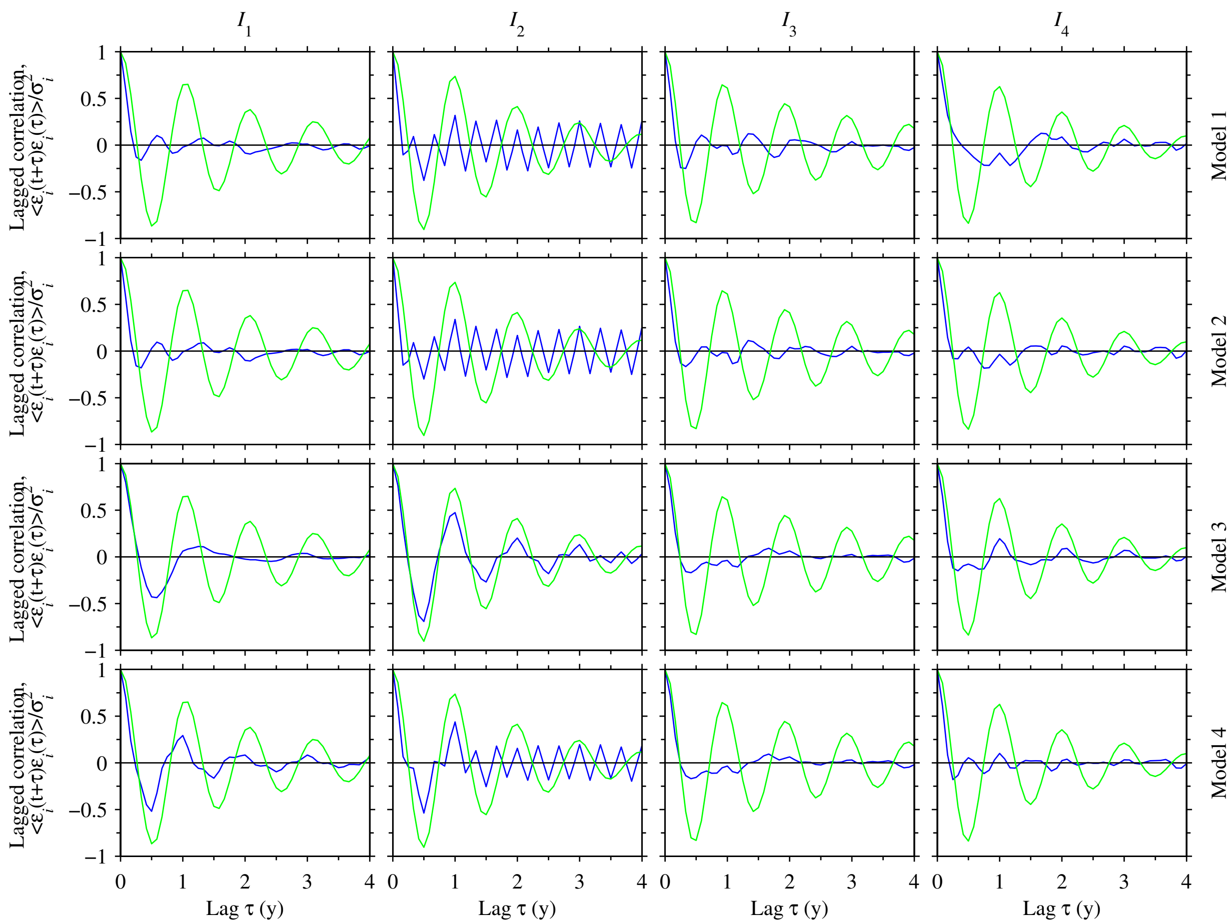

Figure 9: Lagged correlation coefficients of the residuals from models 1–4 in Tables 1–4 (blue lines). Shown for reference are the lagged correlation coefficients of the corresponding response variables, $I_1, \ldots, I_4$ (green lines).