Hierarchical Structure of the Madden-Julian Oscillation in Infrared Brightness Temperature Revealed through Nonlinear Laplacian Spectral Analysis

Dimitrios Giannakis
Courant Institute of Mathematical Sciences
New York University
New York, USA
dimitris@cims.nyu.edu

Wen-wen Tung
Department of Earth, Atmospheric, & Planetary Sciences
Purdue University
West Lafayette, USA
wwtung@purdue.edu

Andrew J. Majda
Courant Institute of Mathematical Sciences
New York University
New York, USA
jonjon@cims.nyu.edu

Abstract—The convection-coupled tropical atmospheric motions are highly nonlinear and multiscaled, and play a major role in weather and climate predictability in both the tropics and mid-latitudes. In this work, nonlinear Laplacian spectral analysis (NLSA) is applied to extract spatiotemporal modes of variability in tropical dynamics from satellite observations. Blending qualitative analysis of dynamical systems, singular spectrum analysis (SSA), and spectral graph theory, NLSA has been shown to capture intermittency, rare events, and other nonlinear dynamical features not accessible through classical SSA. Applied to 1983–2006 satellite infrared brightness temperature data averaged over the global tropical belt, the method reveals a wealth of spatiotemporal patterns, most notably the 30–90-day Madden-Julian oscillation (MJO). Using the Tropical Ocean Global Atmosphere Coupled Ocean Atmosphere Response Experiment period as an example, representative modes associated with the MJO are reconstructed. The recovered modes augment Nakazawa’s classical hierarchical structure of intraseasonal variability with intermediate modes between the fundamental MJO envelope and super cloud clusters.

I. INTRODUCTION

Satellite imagery has been used to study convection-coupled tropical disturbances since the 70s. Substantial advancements in the understanding of tropical waves have been made through linear theories and diagnostics guided by these theories (e.g., [1]). However, convection-coupled tropical motions are highly nonlinear and multiscaled. The most notable example is the Madden-Julian oscillation (MJO, e.g., [2]). The eastward-propagating MJO has gross scales in the 30–90-day intraseasonal time range and zonal wavenumber of order 2–4 in space. It dominates the tropical predictability in the subseasonal time scales, exerting global influences through tropical and extratropical interactions, affecting high-impact weather and climate variability, and fundamentally interfacing the short-term weather prediction and long-term climate projections [3].

Conventional methods for extracting the MJO signals from observations and models are linear, including linear band-pass filter, regression, and empirical orthogonal functions (EOFs) [4]. Nevertheless, linear filtering of a nonlinear, chaotic system impedes fundamental understanding of the system [5]. Theory development has suggested that the MJO is a nonlinear oscillator [6], [7]. With a nonlinear temporal filter, the observed MJO appears to be a stochastically driven chaotic oscillator [8]. In this paper, nonlinear Laplacian spectral analysis (NLSA, [9], [10], [11]) is applied to extract the spatiotemporal patterns of the MJO and its subcomponents from satellite infrared brightness temperature data averaged over the tropical belt. It is the first time that the MJO pattern emerges organically through machine learning.

The plan of this paper is as follows. In Section II, we describe the dataset used in this study. Section III contains an overview of NLSA algorithms. We present and discuss our results in Section IV, and conclude in Section V.

II. DATASET DESCRIPTION

The Cloud Archive User Service (CLAUS) Version 4.7 multi-satellite infrared brightness temperature (denoted $T_b$) [12] is used for this study. Brightness temperature is a measure of the earth’s infrared emission in terms of the temperature of a hypothesized blackbody emitting the same amount of radiation at the same wavelength [∼10–11 µm in the CLAUS data]. It is a highly correlated variable with the total longwave emission of the earth. In the tropics, positive (negative) $T_b$ anomalies are associated with reduced (increased) cloudiness. The global CLAUS $T_b$ data are on a 0.5° longitude by 0.5° latitude fixed grid, with three-hour time resolution from 00 UTC to 21 UTC, spanning July 1, 1983 to June 30, 2006. $T_b$ values range from 170 K to 340 K with approximately 0.67 K resolution.

The subset of the data in the global tropical belt between 15°S and 15°N was averaged to create a longitude-time dataset sampled at $d = 720$ gridpoints and $s = 67,208$ temporal snapshots. Prior to spatial averaging, the missing data (less than 1%) were filled via linear interpolation in time. The portion of the data for the period of the Tropical Ocean Global Atmosphere Coupled Ocean Atmosphere Response Experiment (TOGA COARE, November 1992 to
February 1993), which is studied in Section IV-B, is shown in Figure 1. There, and in Figures 5 and 6, time increases from bottom to top, so that lines of positive (negative) slope correspond to eastward (westward) speeds of propagation.

III. NLSA ALGORITHMS

Nonlinear Laplacian spectral analysis [9], [10], [11] is a method for extracting spatiotemporal patterns from high-dimensional time series that blends ideas from the qualitative analysis of dynamical systems [13], [14], singular spectrum analysis (SSA) [15], [16], and spectral graph theory for machine learning [17], [18]. Given a dataset consisting of $s$ samples of a $d$-dimensional time series $x(t) = (x_1(t), \ldots, x_d(t))$ taken uniformly at times $t_i = i \Delta t$ with $i \in \{1, \ldots, s\}$, the objective of NLSA is to produce a decomposition of the form

$$x(t) \approx \sum_{k=1}^{l} x^k(t), \quad x^k(t) \in \mathbb{R}^d,$$

(1)

where each spatiotemporal pattern $x^k(t)$ reveals meaningful information about processes operating in the data. In Section IV, $x(t)$ will be the latitude-averaged CLAUS $Tb$ data, i.e., a slice along the latitude axis of Figure 1.

When $x(t)$ is generated by a complex, nonlinear dynamical system, physically-important processes are not necessary those that carry large variance, and therefore captured by SSA and related algorithms [19], [20]. NLSA attempts to correct that deficiency by describing temporal patterns in terms of natural orthonormal basis functions on the nonlinear data manifold, $M$, whose properties depend on the local geometric structure of the data, rather than extrinsic effects related to how the dataset was embedded in ambient space. The main steps employed in the construction are (1) time-lagged embedding; (2) calculation of temporal basis functions through graph-theoretic algorithms with time-adaptive Gaussian weights; (3) singular value decomposition (SVD) of linear maps acting on scalar functions on $M$; (4) estimation of the temporal space dimension; (5) projection from lagged-embedding space to physical space. Below, we provide a high-level description of each step, referring the reader to Refs. [10], [11] for further details and pseudocode.

A. Time-lagged embedding

The first step in NLSA involves choosing a lagged-embedding window $\Delta t = q \delta t$ with $q$ a positive integer, and embedding the dataset in a space of dimension $n = dq$ (so-called embedding space) via the method of delays,

$$x(t_i) \mapsto X_i = (x(t_i), x(t_i - \delta t), \ldots, x(t_i - (q-1) \delta t)).$$

(2)

This procedure helps recover some of the phase-space information lost by partial observations [13], [14], [16]; a situation widely encountered in geophysical applications. Moreover, because each point in $\mathbb{R}^n$ corresponds to a trajectory of length $\Delta t$ in physical space $\mathbb{R}^d$, lagged embedding allows one to represent propagating structures (such as the MJO) via individual vectors in embedding space [21]. In general, several considerations come into play in the selection of $\Delta t$, including characteristic timescales of the phenomena under study, the total timespan of the available data, and the influence of external factors (e.g., solar forcing) and other non-Markovian effects. We will return to these issues in Section IV-A. Hereafter, we use the notation $X_{ij}$ to represent the $i$-th component (gridpoint value) of sample $X_j$ in embedding space, and $X = [X_{ij}]$ to represent the $n \times s'$ data matrix with $s' = s - (q+1)$.

B. Temporal basis functions

Next, consider the two alternative ways of interpreting the dataset in embedding space:

1) A linear map from $\mathbb{R}^s$ to $\mathbb{R}^n$ acting via standard matrix multiplication on vectors $f = (f_1, \ldots, f_s)$,

$$f \mapsto y = Xf, \quad f \in \mathbb{R}^s, \quad y \in \mathbb{R}^n.$$

(3)

2) A graph $G$ embedded in $\mathbb{R}^n$ with vertices $\mathcal{V} = \{X_1, \ldots, X_{s'}\}$ and edges $\mathcal{V} \times \mathcal{V}$ equipped with a weight matrix $W$ [determined via Eq. (10) ahead].
In SSA and related algorithms [15], [16], the spatial and temporal patterns associated with the signal $X_i$ are determined through the singular value decomposition (SVD) of $X$,

$$X = U \Sigma V^T,$$

where $U$ and $V$ are orthogonal matrices of dimension $n \times n$ and $s' \times s'$, respectively, and $\Sigma$ a diagonal matrix with nonnegative diagonal entries. In Eq. (4), the $j$-th column of $V$ gives rise to a function of time [a principal component (PC)] $v_j(t_i) = V_{ij}$. The corresponding spatial pattern, $U_j = (U_{1j}, \ldots, U_{nj})$, referred to in this context as an extended EOF (EEOF) [16], [21], can be visualized as a spatiotemporal process $u_j(\tau_i)$ in $\mathbb{R}^d$ of time duration $\Delta t$, viz.

$$u_j(\tau_i) = (U_{d+(i-1)+1,j}, \ldots, U_{d+j}), \quad \tau_i = (i-1) \delta t. \quad (5)$$

From the point of view of 2), each temporal pattern can be associated with a scalar function $\tilde{v}_j$ on the vertex set of $G$ given by $\tilde{v}_j(X_i) = v_j(t_i)$. Conceptually, $\tilde{v}_j$ can be thought of as a scalar function on a nonlinear manifold $M$ covered densely by $G$ as $s' \to \infty$, even though practical high-dimensional datasets will be far from that continuum limit.

One would expect the prominent dynamical processes operating in the data to be describable in terms of well-behaved scalar functions $v_j(X_i)$, collectively playing the role of a set of coordinates (or features) which vary smoothly on $M$. The benefits of working with coordinates tailored to the intrinsic geometry of the data have been established in nonlinear algorithms for machine learning (e.g., [22], [17], [18]), but there is no provision in the classical formulation of SSA to enforce geometrical regularity of the temporal patterns on the data manifold. Indeed, because the $v_j(t)$ from Eq. (4) correspond to linear projections onto the principal axes of the covariance matrix, $XX^T$, those functions can develop high-frequency oscillations and other features which are not related to the intrinsic geometry of $M$ (e.g., if the embedding of the data in $\mathbb{R}^n$ produces folds). However, despite their ability to produce low-dimensional descriptions of the data, nonlinear algorithms are prone to overfitting [23], and preclude an unambiguous “lifting” operation from the reduced coordinates to data space [24].

NLSA algorithms address these issues by retaining aspects from both 1) and 2) above, employing spectral analysis of linear maps to evaluate spatial and temporal patterns, while requiring those maps to act on spaces of well-behaved scalar functions on the nonlinear data manifold. Specifically, the temporal patterns in NLSA are required to lie in low-dimensional function spaces $V_l$ spanned by the leading $l$ eigenfunctions of a graph Laplace-Beltrami operator $\Delta$, i.e.,

$$V_l = \text{span}\{\phi_0, \ldots, \phi_{l-1}\}, \quad \dim V_l = l, \quad (6)$$

with

$$\sum_{k=1}^{s'} P_{jk} \phi_k(X_k) = \lambda_i \phi_j(X_i), \quad (7)$$

$$\Delta = P - I, \quad 1 = \lambda_0 < \lambda_1 \leq \lambda_2 \leq \cdots. \quad (8)$$

In Eq. (7), $P$ is an $s' \times s'$ Markov matrix ($\sum_j P_{ij} = 1$) associated with the weight matrix $W$ of $G$. The invariant measure $\mu$ of $P$, $\sum_j \mu_j P_{ij} = \mu_j$, gives rise to a weighted inner product

$$\langle f_1, f_2 \rangle = \sum_{i=1}^{s'} \mu_i f_1(X_i) f_2(X_i). \quad (9)$$

between scalar functions defined on the vertex set $V$ of $G$. It is a standard result that the $\phi_j$ from Eq. (7) are orthogonal with respect to the inner product in Eq. (9). Because the generator $\Delta$ of $P$ converges under certain conditions to the manifold Laplacian, hereafter we refer to $\mu$ as a Riemannian measure, and interpret $\mu_j$ as the volume occupied by sample $X_i$ in the data manifold.

Here, we construct $P$ using the Diffusion Map (DM) algorithm of Coifman and Lafon [18], modified by time-dependent Gaussian weights

$$W_{ij} = \exp\left(-\frac{|X_i - X_j|^2}{\epsilon \xi_i \xi_j}\right), \quad (10)$$

where $\xi_i = ||X_i - X_{i-1}||$ is a local velocity in embedding space. Because the Riemannian measure in the DM family of algorithms is given by

$$\mu_i \propto \sum_{j=1}^{s'} \frac{W_{ij}}{\left(\sum_{k=1}^{s'} W_{ik}\right)^\alpha}, \quad (11)$$

for some parameter $\alpha \in \mathbb{R}$, states with large $\xi_i$ acquire larger Riemannian measure than in the uniform-$\xi$ case. This property enhances the skill of the algorithm to detect rapid transitions and rare events, which can be crucial for reduced dynamical modeling through Galerkin projections [10]. Moreover, scaling the squared pairwise distances by $\xi_i \xi_j$ alleviates the need of tuning $\epsilon$ by hand. In the applications presented here and in Refs. [10], [11], [25] we have found that skillful and robust results can be obtained by setting $\epsilon = 1$ or $O(1)$. Locally-scaled kernels have also been used in other works in the literature (e.g., [26], [27], [28]), with the difference that the $\epsilon_i$ are determined here on the basis of temporal rather than spatial nearest neighbors.

C. Singular value decomposition

Having established the temporal spaces $V_l$, the next step in the NLSA is to form a family of linear maps $A^l : V_l \mapsto \mathbb{R}^n$, which are represented by $n \times l$ matrices with elements

$$A^l_{ik} = \sum_{j=1}^{s'} \mu_j X_{ij} \phi_k(X_j). \quad (12)$$
That is, a function $f = \sum_{k=1}^{l} c_k \phi_k$ in $V_i$ is mapped to $y = A^i(f)$ with $y = (y_1, \ldots, y_n)$ and $y_l = \sum_{k=1}^{l} A^i_k c_k$. These linear maps generalize the corresponding map for SSA in Eq. (3) to take into account the nonlinear manifold structure of the data. The spatial and temporal patterns associated with $A^i$ follow in analogy with Eq. (4) by performing the SVD

$$A^i = U \Sigma V^T$$

where $U$ and $V$ are $n \times n$ and $l \times l$ orthogonal matrices, and $\Sigma = \text{diag}(\sigma_1^i, \ldots, \sigma_l^i)$ a diagonal matrix of nonnegative singular values. Here, the matrix elements of $V$ are expansion coefficients for elements of the function space $V_i$ in the $\phi_i$ basis, corresponding to functions of time

$$v_j(t_i) = \sum_{k=1}^{l} v_{kj} \phi_{k-1}(X_i),$$

which are the NLSA analogs of the PCs in classical SSA. By the orthogonality properties of the $\phi_i$ basis functions, the $v_j$ are orthogonal with respect to the inner product in Eq. (9). Below, we use the shorthand notations $U_i = (U_{1i}, \ldots, U_{ni})$ and $v_i = (v_i(t_1), \ldots, v_i(t_n))$ to represent the $j$-th spatial and temporal patterns, respectively.

**D. Setting the temporal space dimension**

Heuristically, the parameter $l$ controls the lengthscale on the data manifold resolved by the eigenfunctions spanning $V_i$. Working at large $l$ is desirable, since the lengthscale on $M$ resolved by the eigenfunctions spanning $V_i$ generally becomes smaller as $l$ grows, but the sampling error in the graph-theoretic eigenfunctions $\phi_i$ increases with $l$ for a fixed number of samples $s'$. In other words, $\phi_i$ will generally depend more strongly on $s'$ for large $l$, resulting in an orfit of the discrete data manifold. A useful way of establishing a tradeoff between improved resolution and risk of overfitting is to monitor a relative spectral entropy $D_l$, measuring changes in the energy distribution among the modes of $A^i$ as $l$ grows [11]. This measure is given by the formula

$$D_l = \sum_{i=1}^{l} p_i^{l+1} \log(p_i^{l+1}/\pi_i^{l+1}),$$

with $p_i^l = \sigma_i^l/(\sum_i^l \sigma_i^l)$, $p_i^{l+1} = \sigma_i^{l+1}/(\sum_i^{l+1} \sigma_i^{l+1})$, and $(\sigma_1^l, \ldots, \sigma_{l-1}^l, \sigma_l^l) = (\sigma_1^{l+1}, \ldots, \sigma_{l-1}^{l+1}, \sigma_l^{l+1}).$ The appearance of qualitatively new features in the spectrum of $A^i$ is accompanied by spikes in $D_l$ (see Figure 2), and therefore a reasonable truncation level is the minimum $l$ beyond which $D_l$ settles to small values.

**E. Projection to physical space**

The final step in the NLSA pipeline is to construct the spatiotemporal patterns $x^k(t)$ in Eq. (1) associated with the corresponding singular vectors and values, $\{U_k, v_k, \sigma_k\}$, of the $A^i$ map in Eqs. (13) and (14). Because $A^i$ is a linear map, this procedure is significantly more straightforward and unambiguous than in methods based on nonlinear mapping functions (e.g., [23], [24]), and consists of two steps: (1) Compute an $n \times s'$ matrix $X^k$ containing the $k$-th spatiotemporal pattern in embedding space, $X^k = U_k \sigma_k v_k^T$. (2) Decompose each column of $X^k$ into $q$ blocks $\hat{x}_{ij}$ of size $d$, and take the average over the lagged embedding window,

$$x^k(t_i) = \sum_{j=1}^{\min(q,i)} \hat{x}_{j,i-j+1}/\min(q,i).$$

This leads to $s$ samples in $d$-dimensional physical space, completing the decomposition in Eq. (1).

**IV. RESULTS AND DISCUSSION**

Nakazawa [29], [30] coined the phrase “hierarchy of intraseasonal variations” while studying the structure of the MJO using $Tb$ and other data in the 1980s. In his schematics of the hierarchy, the $O(10^3)$ km MJO spatial scale moves eastward with a phase speed of 4–8 $\text{m s}^{-1}$. Within the envelope of the MJO, there are both eastward and westward disturbances. The eastward “super clusters” move at speeds of 10–15 $\text{m s}^{-1}$, with scales of $O(10^3)$ km in space and $O(10)$ d in time, and comprise of smaller westward moving “cloud clusters” with 1–2-day lifetimes. The super clusters were later identified as the convection-coupled Kelvin waves [31] and the cloud clusters as inertia-gravity waves [32]. Inspired by Nakazawa’s schematics, below we show the hierarchy of modes revealed by NLSA and a reconstruction of the MJO observed during TOGA COARE.

**A. Spatiotemporal patterns revealed by NLSA**

We have applied the NLSA algorithm described in Section III using an embedding window spanning $\Delta t = 64$ d. This amounts to an embedding space dimension $n = q d = 368,640$ for the $\delta t = 3$ h sampling interval ($q = \Delta t/\delta t = 512$) and 0.5° resolution of our dataset (see Section II). This choice of embedding window was motivated from our objective to resolve propagating structures such as the MJO with intraseasonal ($30–90$ d) characteristic timescales. Unlike conventional approaches (e.g., [21], [33], [34]), neither bandpass filtering nor seasonal partitioning was applied to the CLAUS dataset prior to analysis using NLSA. All calculations presented here were implemented in Matlab, and executed on a 32-core Linux server with 128 GB RAM.

For the calculation of the graph Laplace-Beltrami eigenfunctions and Riemannian measure in Eqs. (7) and (11) we computed the pairwise distances of the data in embedding space using brute force, and evaluated the weight matrix in Eq. (10) using $\epsilon = 1.8$ and retaining nonzero weights for 5,000 nearest neighbors per datapoint. We set the DM
For consistency with the TOGA-COARE reconstruction in Section IV-B, the left-hand panels show the time series from October 1992 to September 1993. The right-hand panels are FFT results applied to the entire length of $v_k(t)$. Figure 4a shows the annual cycle, $v_1(t)$, as seen in the time series as well as in the frequency spectrum. Its corresponding spatial pattern, $u_1(\tau)$ [see Eq. (5)], in Figure 5 is nearly constant within the embedding window $\Delta t$. The annual cycle and longer time variability are also found in $v_2(t)$ and $u_2$ (not shown). Modes 1 and 2 are a complementary pair, as suggested by Figure 3. Together, they explain the seasonal and climate variability of cloudiness in the tropical belt.

Figures 4(b,c) and 5(b,c) are identified as the MJO modes. They feature broad peaks centered around 60 days in their frequency spectra. The spatial patterns, $u_3$ and $u_4$, are in quadrature, indicating that together they represent the propagation of the pattern of the MJO convective systems (roughly, of global wavenumber two) over the Indian Ocean-western Pacific sector. $u_3$ and $u_4$ also exhibit spatial structure over the Maritime Continent between 100° and 150° E. Along with these fundamental MJO modes, the NLSA spectrum contains a wide variety of spatiotemporal patterns with intraseasonal variability, some of which are discussed in the TOGA COARE reconstruction below.

Lastly, Figures 4(d) and 5(d) show one of the diurnal modes, with a dominant peak over once-per-day frequency in the spectrum of $v_5(t)$ and clearly diurnally-repeating.

Parameter $\alpha$ to unity, corresponding to the Laplace-Beltrami normalization [18]. The resulting spectral entropy $D_l$, computed via Eq. (15) and normalized to the unit interval by applying the transformation $1 - \exp(-2D_l))^{1/2}$ [35], is shown in Figure 2. As described in Section III-D, $D_l$ exhibits a series of spikes as $l$ increases from small to moderate values ($l \sim 20$), which correspond to qualitatively new spatiotemporal patterns entering in the spectrum of $A^l$. Eventually, $D_l$ settles to small values for $l \gtrsim 25$. On the basis of the results in Figure 2, hereafter we set the temporal space dimension in Eq. (6) to $l = 27$. The singular values $\sigma_i^l$ of the corresponding $A^l$ linear map from Eq. (13) are displayed in Figure 3. Note that the matrix size of $A^l$ is $n \times l = 368,640 \times 27$. In contrast, the data matrix $X$ used in SSA is dimensioned $n \times s' = 368,640 \times 66,696$. With the available resources, we were not able to perform SSA on the full dataset to compare against our NLSA-based results.

Representative temporal and spatial patterns are discussed with Figures 4 and 5. Figure 4 shows the temporal patterns $v_k(t)$ with $k \in \{1, 3, 4, 5\}$ and their frequency spectra. For consistency with the TOGA-COARE reconstruction in
patterns throughout the length of the embedding window in the spatial pattern $u_5$. These diurnal cycles are most conspicuous over the tropical Africa (30° E) and America (60° W), although they can also be seen over the Maritime Continent. As indicated in Figure 3, this mode and $v_6$ are a twofold-degenerate pair. Upon reconstruction, they reveal the standing diurnal convective events over tropical Africa and America (not shown).

B. Reconstruction of the TOGA COARE period

Two complete MJO events were observed during the TOGA COARE (e.g., [36]). Figure 1 shows the longitude-time section of the CLAUS $T_b$ averaged in the global tropical belt during the TOGA COARE. The MJO events, marked by two distinct envelopes of super clusters distinguished by cold $T_b$, propagated eastward from the Indian Ocean to the date line. The first event started near 75° E in late November, subsequently crossed the Maritime Continent around 100°–150° E, then disappeared near 170° W around January 10. The second event, being slightly faster than the first, started around January 5, and reached the central Pacific in early February. A third event started in March, after the end of the TOGA COARE. The TOGA COARE period was coincident with the amplifying phase of an El Niño event; therefore, the super clusters propagated further east beyond the date line, where during normal years the cold sea surface temperature is not conducive to deep convection. The eastward propagating speed of the MJO events was $\sim 4$–$5$ ms$^{-1}$. The convective systems around the Maritime Continent were especially complicated before, during, and after the passages of the MJO. In addition, two regions of apparently standing convection over equatorial Africa and America were observed.

Figure 6 shows the reconstruction of MJO super clusters and a few related convective disturbances using the modes identified through NLSA. The algorithm detected a wide variety of spatiotemporal disturbances during the TOGA COARE period. A few significant ones are discussed below.

Figure 6(a) is the MJO reconstruction, which captures the succinct features of the propagating envelope of super clusters, including the initiation of enhanced cloudiness (hence cold anomalies) over the Indian Ocean, the passage over the Maritime Continent, and the arrival and demise near the date line. The two reconstructed MJO events propagate at a speed of order 4–5 ms$^{-1}$.

Figure 6(b) is reconstructed from a higher-order mode (11, see Figure 3). It exhibits an eastward propagating disturbance with a speed of $\sim 7$–$8$ ms$^{-1}$. It mainly comprises of two deep convective systems, each with a zonal scale of order 500 km, centered around 90° E and 120° E, respectively. This mode may indicate the convective activity around the Maritime Continent, which is modulated by the passing of the MJO. The superposition of this and the MJO modes may occasionally result in a standing convective activity around the Maritime Continent. Note that, upon examination of a longer time period than TOGA COARE, this mode can have significant amplitude when the MJO mode does not.

An intriguing pattern, reconstructed from mode 14, is shown in Figure 6(c), in which the major convective systems...
appears to be split into two by the southeast Asian archipelagos. The one over the Indian ocean is out of phase with the MJO mode in the region. The other is over the western Pacific and is roughly in phase with the MJO. Upon first look, both systems appear to be propagating eastward at the same speed of the MJO. However, the dichotomy of the spatial pattern also suggests westward-propagating signals. Unlike mode 11, the amplitude of this mode is locked in with that of the MJO mode. A conjecture, before further analysis of the dynamical fields associated with this mode, is that it is a Gill-type pattern [37], [38].

V. CONCLUSIONS AND FUTURE WORK

The NLSA algorithm applied to CLAUS $Tb$ data averaged over the global tropical belt reveals a hierarchy of spatiotemporal modes, providing an augmented picture of Nakazawa’s [29], [30] paradigm. The recovered modes are likely collective patterns of multiscale cloud clusters and other disturbances. A physical interpretation of the hierarchy of modes occurring concurrently with the fundamental MJO modes should lead to an improved understanding of the strengths and weaknesses of the algorithm. Another fruitful line of research would be to simultaneously include $Tb$ and zonal wind data in the analysis to produce NLSA-based multivariate MJO indices (e.g., [34]). We plan to pursue these topics in future work.

ACKNOWLEDGMENT

We thank Duane Waliser for stimulating discussions. AJM and DG acknowledge support from ONR DRI grants N25-74200-F6607 and N00014-10-1-0554.

REFERENCES


