Research article

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Initiation and termination of intraseasonal oscillations in nonlinear Laplacian spectral analysis-based indices

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Abstract: We present a statistical analysis of the initiation and termination of boreal winter and boreal summer intraseasonal oscillations (ISOs). This study uses purely convection (infrared brightness temperature) data over a 23-year time interval from 1984–2006. The indices are constructed via the nonlinear Laplacian spectral analysis (NLSA) method and display high intermittency and non-Gaussian statistics. We first define primary, terminal, and full events in the NLSA-based indices, and then examine their statistics through the associated two-dimensional phase space representations. Roughly one full event per year was detected for the Madden-Julian oscillation (MJO), and 1.3 full events per year for the boreal summer ISO. We also find that 91% of the recovered full MJO events are circumnavigating and exhibit very little to no retrograde (westward) propagation. The Indian Ocean emerges as the most active region in terms of both the onset and decay of events, however relevant activity occurs over all phases, consistent with previous work.

Keywords: Nonlinear Laplacian spectral analysis (NLSA), Madden-Julian oscillation, tropical intraseasonal oscillations

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1 Introduction

Intraseasonal oscillations (ISOs) are large-scale modes of tropical variability playing a key role in the global climate system through extratropical interactions and feedback (Lau and Waliser, 2011; Zhang, 2013). The dominant boreal winter ISO is the well-known Madden-Julian oscillation (MJO) (Madden and Julian, 1971, 1972), a 30–90-day eastward-propagating pattern of convective activity with zonal wavenumber 1–4. The MJO most commonly initiates in the western Indian Ocean and propagates over the Maritime Continent into the western Pacific at a speed of approximately 5 ms$^{-1}$ (Zhang, 2005). On the other hand, the dominant boreal summer ISO (BSISO) has a northeastward-propagating pattern that tends to initiate in the Indian Ocean and propagate towards India and southeast Asia (Wang and Rui, 1990; Kikuchi et al., 2012). As BSISO propagates into the Indian monsoon region at a frequency of 30–60 days, it influences significantly the monsoon’s onset and active/break phases (Goswami, 2011).

Many theories have been proposed in the literature to explain key features of the ISOs, such as onset, strength, and decay with a strong emphasis on the MJO and lately increasingly on the BSISO. In particular, various studies have found links between MJO initiation and other atmospheric variables such as low-level heating and moisture anomalies (Hendon and Salby, 1994; Khouider and Majda, 2006), the organization of planetary-scale wind anomalies into a wavenumber-1 pattern (Straub, 2013), or diabatic heating and precipitation anomalies (Ling et al., 2014). Different precursor conditions, particularly low-level moisture anomalies, have been related to MJO termination (Stachnik et al., 2015). The strength of the MJO has also been linked to the El Niño Southern oscillation (ENSO) (Zhang and Gottschalck, 2002; Hendon et al., 2007), with strong MJO activity often observed during ENSO-neutral years, and weak or absent MJO activity typically associated with strong El Niño or La Niña episodes. In the Pacific, strong MJO activity is often observed 6–12 months prior to the peaks of El Niño episodes, e.g., the strong 1996-1997 MJO preceding the strong 1997-1998 El Niño (Kessler, 2011). Yet, despite their strong impact on predictability (Waliser, 2011), ISOs are generally simulated poorly by current general circulation models (GCMs) (Hung et al., 2013).

In addition to theories explaining ISO features, there is also a need for indices that quantify their strength for monitoring and prediction purposes. Since ISOs are propagating patterns of large areas of either enhanced or suppressed convection accompanied by lower- and upper-level atmospheric circulation anomalies, construction of ISO indices typically takes into account convection and/or circulation data. Proxies most often used for convective activity are either cloudiness
(outgoing longwave radiation (OLR) and brightness temperature ($T_b$)) or rainfall data. Circulation is most commonly represented by lower- and upper-level zonal winds, but also streamfunctions and velocity potentials. Many data-based indices have been proposed to describe ISO activity, mainly for the MJO (Wheeler and Hendon, 2004; Matthews, 2008; Straub, 2013; Kiladis et al., 2014), but also for the BSISO (Lee et al., 2013; Kikuchi et al., 2012; Székely et al., 2015). The techniques range from spacetime filtering methods (Wheeler and Kiladis, 1999; Kiladis et al., 2005; Kikuchi and Wang, 2010) to empirical orthogonal functions (EOFs) (Lo and Hendon, 2000; Maloney and Hartmann, 1998; Kessler, 2001; Wheeler and Hendon, 2004; Kikuchi et al., 2012; Ventrice et al., 2013; Kiladis et al., 2014), as well as hybrid filtering–EOF approaches (Roundy and Schreck, 2009). Among the multitude of indices, the real-time multivariate MJO (RMM) index (Wheeler and Hendon, 2004) is the most common measure of ISO activity used all year-round, both for boreal winter and boreal summer activity. RMM is a combined measure of the first two EOFs of bandpass-filtered, and equatorially averaged OLR and 200hPa and 850hPa zonal wind data. In spite of the normalization of both the cloudiness and circulation components, the index is dominated by the circulation component (Straub, 2013). In particular, the bivariate correlation between full RMM and wind-only RMM is 0.99.

Recently, Székely et al. (2015) (hereafter, S15) developed MJO and BSISO indices based purely on convection data. These indices were constructed using the nonlinear Laplacian spectral analysis (NLSA) (Gianakis and Majda, 2012, 2013, 2014), a kernel eigendecomposition technique for high-dimensional (spatiotemporal) data combining ideas from machine learning and Takens delay-coordinate maps of dynamical systems. Compared to traditional approaches such as EOF and extended EOF analysis, NLSA provides superior timescale separation with no preprocessing of the input data such as seasonal partitioning, equatorial averaging, or bandpass filtering. The output of NLSA consists of a hierarchy of temporal and spatiotemporal modes that exist at different timescales. On the intraseasonal scale, NLSA outputs two distinct pairs of eigenmodes for the dominant boreal winter and boreal summer ISOs [S15]. This feature has proven particularly useful in the spatiotemporal reconstructions which reflect the distinct propagating patterns of the ISOs, i.e., eastward propagation for MJO vs. northeastward propagation for BSISO.

S15 built indices for MJO and BSISO from the NLSA ISO eigenfunction pairs active in boreal winter and boreal summer, respectively. Compared to EOF-based approaches, NLSA indices have higher discriminating power between the boreal winter and boreal summer ISOs, and are characterized by strong intermittency with seasonally dependent periods of quiescence and activity. These properties are an outcome of NLSA’s nonlinear kernel which is designed to cap-
ture patterns generated by complex dynamical systems. Moreover, the spatial patterns associated with the NLSA kernel eigenfunctions are not subjected to an orthogonality constraint.

In this paper, we extend the study of S15 to objectively (i.e., without preprocessing the input data) analyze the onset, evolution, and decay of the MJO and BSISO as represented by the NLSA indices. In particular, we define primary and terminal events with respect to predefined thresholds similarly to previous works based on EOFs (Matthews, 2008; Straub, 2013; Stachnik et al., 2015), and study the statistics of these events. Due to the strong MJO-BSISO discrimination, clean frequency content, and heavy-tailed distributions of the NLSA ISO indices, we are able to use lower amplitude thresholds, and thus detect earlier the onset of events and avoid premature terminations. The climatology, i.e., the locations and frequency of primary and terminal events, is similar to that presented in previous works (Matthews, 2008; Straub, 2013; Stachnik et al., 2015), but there are many differences in the details of individual events, especially in terms of the number of events that complete a full rotation (eight phases) in the two-dimensional phase space. In particular, the number of full events detected via NLSA is significantly smaller than in the case of RMM, and most MJOs are circumnavigating events with approximately one event occurring each year.

The paper is organized as follows. Section 2 describes the infrared brightness temperature data used in the analysis. The MJO and BSISO indices constructed using NLSA eigenfunctions are presented in Sect. 3, followed by examples and a study of the climatology of primary, terminal, and full events for each of the ISOs. Section 4 presents a detailed comparison of the two indices and the differences in their features followed by a comparison with other state-of-the-art indices for ISO analysis. The paper ends with conclusions in Sect. 5. Two appendices with additional details on the initiation and termination of individual ISO events (Appendix A) and the influence of the threshold values on event detection (Appendix B) are also included.

2 Data

Satellite infrared brightness temperature ($T_b$) data from the Cloud Archive User Service (CLAUS) (Hodges et al., 2000) is used to extract pure cloudiness ISO signals and build MJO and BSISO indices. The data covers a time period of 23 years from January 1, 1984 to June 30, 2006. In the tropics, positive (negative) $T_b$ anomalies are associated with reduced (increased) cloudiness and are a good proxy for tropical convection. The data is sampled over the tropical belt from
15°S to 15°N with a resolution of 1° (in both longitude and latitude) generating 2D samples with \( n_{\text{long}} = 360 \) longitude and \( n_{\text{lat}} = 31 \) latitude gridpoints. Observations are collected at an interval of \( \delta t = 6 \text{h} \), producing a dataset with \( s = 32,868 \) samples over the 23 years of the CLAUS record.

3 NLSA-based ISO indices

Blending ideas from delay embeddings for dynamical systems (Packard et al., 1980; Takens, 1981; Broomhead and King, 1986; Sauer et al., 1991) and spectral graph theory from machine learning (Belkin and Niyogi, 2003; Coifman and Lafon, 2006), nonlinear Laplacian spectral analysis (NLSA) (Giannakis and Majda, 2012, 2013, 2014) identifies temporal and spatiotemporal patterns of interest in high-dimensional time series. NLSA extracts a set of temporal modes (which can be thought of as nonlinear principal components) through the eigenfunctions of a Laplace-Beltrami operator tailored to the nonlinear geometry and dynamics of the data. These temporal modes form a hierarchy of patterns evolving at different timescales, including interannual signals (e.g., ENSO), the annual cycle and its harmonics, and intraseasonal and diurnal signals (S15; Tung et al., 2014). While in RMM the first three harmonics of the annual cycle and the interannual variability are removed prior to the analysis, NLSA requires no preprocessing of the input data. The fact that the input data are not subjected to bandpass filtering opens up the possibility to explore directly the relationship between intraseasonal modes and other important modes of tropical variability, such as ENSO and the diurnal cycle.

The core of NLSA consists of: 1) time-lagged embedding using the delay method, followed by 2) the calculation of a set of eigenfunctions using kernel methods from machine learning. While RMM uses the principal components (PCs) given by the eigenfunctions of the covariance operator, NLSA employs the eigenfunctions of a discrete diffusion operator. The eigenfunctions of this operator form a natural orthonormal basis set of functions on the nonlinear manifold sampled by the data, providing superior timescale separation (Berry et al., 2013) than what is possible through linear methods. Such patterns carry low variance and may fail to be captured by variance-greedy algorithms such as EOF analysis, yet may play an important dynamical role (Aubry et al., 1993; Giannakis and Majda, 2012). A detailed description of the method, i.e., the construction of the kernel and the computation of the eigenfunctions of the diffusion operator, can be found in Giannakis and Majda (2012, 2013, 2014). S15 used the CLAUS \( T_b \) infrared brightness temperature data sampled every
δt = 3h to extract a hierarchy of signals at different timescales, from interannual to diurnal signals. Here, after performing analyses using two sampling intervals, δt = 3h and δt = 6h, we found that the latter generated slightly cleaner ISO signals, i.e., the intraseasonal modes were less mixed with the diurnal cycle. Because our main focus in this work is the analysis of intraseasonal signals we use the CLAUS T_b dataset with the lower sampling interval, δt = 6h. We mention that in RMM the diurnal cycle does not pose a problem because the observations are sampled once a day.

Let φ_j = (φ_1j, . . . , φ_Sj)^T be the eigenvectors of the diffusion operator constructed from the data, where S is the number of available samples after delay embedding. Each eigenvector corresponds to a temporal mode of variability φ_j(t_i) = φ_ij sampled at times t_i = (i − 1) δt. Figure 1 shows the two pairs of intraseasonal modes for the boreal winter MJO (Fig. 1(a, b)) and the boreal summer BSISO (Fig. 1(c, d)). Their associated Laplace-Beltrami eigenfunctions from NLSA are the pairs {φ_10, φ_11} and {φ_16, φ_17}, which we use to define indices for the MJO and BSISO, respectively. Following Kikuchi et al. (2012) we also construct individual index amplitudes for MJO and BSISO from the eigenfunctions, i.e.,

\[ r_{t}^{MJO} = \sqrt{\phi_{10}^2(t) + \phi_{11}^2(t)}, \quad r_{t}^{BSISO} = \sqrt{\phi_{16}^2(t) + \phi_{17}^2(t)}. \] (1)

Figure 1(e, f) shows time series of the NLSA-based and RMM indices. The NLSA MJO and BSISO eigenfunctions and indices exhibit high intermittency and strong seasonality, with MJO mainly active in December–April and BSISO in June–October. The probability density functions (PDFs) of the ISO eigenfunctions, shown in Fig. 2, are highly non-Gaussian and have fat tails when computed from the year-round data (Fig. 2(a, b)). Their kurtosis (i.e., measure of “tailedness”) values κ are high and exceed significantly the kurtosis value of the Gaussian distribution (κ = 3). This property facilitates the selection of initiation and termination thresholds for ISO events in Sects. 3.1 and 3.2 ahead. In contrast, the RMM series (RMM1 and RMM2) are active all year-round (with the strongest activity taking place in December–May) and have nearly Gaussian statistics (Fig. 2(a, b)) with kurtosis κ = 3.05 for RMM1 and κ = 2.99 for RMM2. Conditioned on the season (Fig. 2(c, d)), the NLSA eigenfunction PDFs match more closely the Gaussian distributions with standard deviations given by the boreal winter and boreal summer subsets, respectively, but still have moderate departures from Gaussianity as measured by their kurtosis values (κ = {3.83, 3.62} for MJO and κ = {3.52, 3.31} for BSISO eigenfunctions).

In what follows, we examine these NLSA indices in detail, focusing on the onset, evolution, and decay of ISO events and their statistics.
Fig. 1. NLSA eigenfunctions for (a, b) boreal winter MJO, and (c, d) BSISO; (e) NLSA-based index amplitudes $r_{t}^{\text{MJO}}$ and $r_{t}^{\text{BSISO}}$ from equation (1); (f) amplitude of the real-time multivariate MJO (RMM) index.
Fig. 2. Probability density functions of the NLSA ISO eigenfunctions compared with the real-time multivariate MJO (RMM) series. The kurtosis of each distribution is also provided. (a) All year-round NLSA MJO eigenfunction $\phi_{10}$ and RMM 1 time series, normalized to have zero mean and standard deviation equal to one. (b) All year-round NLSA BSISO eigenfunction $\phi_{17}$ and RMM 1 time series, normalized as in (a). (c) Boreal winter NLSA MJO eigenfunction $\phi_{10}$ sampled over the boreal winter (December–April) compared against a Gaussian distribution with the same standard deviation. (d) Boreal summer NLSA BSISO eigenfunction $\phi_{17}$ sampled over the boreal summer (June–October) compared against a Gaussian distribution with the same standard deviation. The kurtosis of the NLSA eigenfunctions (year-round and season dependent) exceeds the kurtosis of a Gaussian distribution ($\kappa = 3$), while RMM’s kurtosis very closely matches it ($\kappa = 3.05$).
3.1 Madden-Julian oscillation

On the intraseasonal timescale, the dominant mode of atmospheric variability is the boreal winter MJO. The two-dimensional phase space diagram of the MJO eigenfunctions from NLSA is displayed in Fig. 3(a). This phase space is split into eight phases and its associated composite life cycle is reconstructed in Fig. 3(b). In this phase space, MJO follows a clockwise rotation that corresponds to an eastward propagation in the spatial domain.

Following previous works (Wheeler and Hendon, 2004; Straub, 2013; Stachnik et al., 2015) we define full MJO events as strong persistent events (subject to the criteria described below) initiated and terminated by so-called primary and terminal events, respectively. First, similarly to Matthews (2008), Straub (2013), and Stachnik et al. (2015), we consider that primary events occur at the first day that the MJO index amplitude is greater than or equal to an initiation threshold $r_{MJO}^P$. Once an event initiates, it becomes a candidate for a full MJO as long as its amplitude does not decay below a given termination threshold $r_{MJO}^T$, called a terminal event. The index has a clockwise rotation, i.e., eastward propagation, in the phase space diagram in Fig. 3(a), with little to no westward (retrograde) propagation. An event is considered a full MJO event if it fulfills the following two conditions with respect to the phase space diagram: 1) the index amplitude crosses at least once the threshold of one standard deviation $\sigma$ (here $\sigma = 1.06$), and 2) it propagates through at least four full phases in the phase space, i.e., half a cycle. The latter condition is similar to Straub (2013) and Stachnik et al. (2015), who require an MJO event to complete four phases of the RMM phase space in order to be considered “full”. In Matthews (2008) a full MJO event was required to propagate through all eight phases of the OLR-based phase space diagram, while in Straub (2013) and Stachnik et al. (2015) events that propagated through all eight phases of their respective phase space diagrams were called “circumnavigating” events. In this paper, we adopt the latter definition requiring an event to complete a full cycle in the two-dimensional phase space in order to be considered “circumnavigating”. We emphasize that a circumnavigation in phase space does not necessarily correspond to a circumnavigation in the spatial (geographical) domain. In the case of the MJO, the two types of circumnavigation coincide to a large extent from Phase 1 to Phase 8 as a spatial propagation of the center of convection through the western hemisphere. However, there are no phases associated with the eastern hemisphere in the two-dimensional phase space diagram as the signal transitions directly from Phase 8 back to Phase 1 (Fig. 3). The difference between circumnavigation in phase space and circumnavigation in the spatial domain becomes even more pronounced in the case of BSISO (see Sect. 3.2).
Fig. 3. (a) Two-dimensional phase space diagram for the MJO eigenfunctions from NLSA. The black circle denotes the constant amplitude of one standard deviation. (b) Spatiotemporal reconstructions of convective anomalies associated with each phase of the MJO index as depicted in (a).
Fig. 4. Examples of strong MJOs. (a) 1996–1997 (Kessler, 2011) and (b) 2003–2004.¹ The black circle denotes the constant amplitude of one standard deviation. The 1996–1997 MJO event occurred prior to the strong 1997–1998 El Niño.

Two examples of full MJOs for the winters of 1996–1997 and 2003–2004 are presented in Fig. 4. Both events were documented to be particularly strong. Strong MJOs have been observed prior to strong El Niño events, and this is the case of the 1996–1997 MJO (Kessler, 2011) which is considered to be a precursor for the initiation of the strong 1997–1998 El Niño. The second example depicts the strong MJO that has been observed in December 2003 – January 2004.¹ Figure 4(b) shows a rapid increase in amplitude starting on December 23, 2003 and a rapid decrease in amplitude after one full circumnavigation with the MJO reemerging and continuing until June, but at a lower intensity.

More specifically, a primary event occurs when the amplitude of the MJO index is lower than the initiation threshold \( r_{MJO}^P \) for three consecutive days, followed by an amplitude higher than \( r_{MJO}^P \) for two consecutive days. That is, we identify the time stamp \( t \) that satisfies the two conditions below:

\[
\begin{align*}
  r_{t+\tau}^{MJO} &< r_{P}^{MJO}, & \tau &\in \{-3, -2, -1\} \text{ days}, \\
  r_{t+\tau}^{MJO} &\geq r_{P}^{MJO}, & \tau &\in \{0, 1, 2\} \text{ days}.
\end{align*}
\]

The PDF of \( r_t^{MJO} \), constructed via kernel density estimation, is plotted in Fig. 5 together with the PDF of the RMM index amplitude. The PDF of the NLSA MJO index amplitude has a strong positive skewness, i.e., it carries large mass at

¹ http://www.esrl.noaa.gov/psd/mjo/MJOprimer/
Fig. 5. Probability density function of the NLSA-based MJO index amplitude $r_{MJO}^{t}$. The red and black dashed vertical lines denote the threshold for initiation at $r_{P}^{MJO} = 0.5$ and the threshold for active MJOs at one standard deviation $\sigma = 1.06$. In comparison, the PDF of the RMM index amplitude more closely resembles a Gaussian distribution.

high values of $r_{MJO}^{t}$. As stated above, we refer to the $r_{P}^{MJO} = \sigma$ threshold as the stage where MJO enters the active phase. By visual inspection of the $r_{t}^{MJO}$ time series (see Fig. 7 ahead), it appears that MJO initiates well in advance before reaching its active phase and therefore we choose the initiation threshold to be lower than the active phase threshold, here $r_{P}^{MJO} = 0.5$ (Fig. 5). We performed robustness tests for the initiation threshold with values in the range 0.3 – 0.8 and the climatology of primary events is relatively stable (Fig. 6(a)).

An event that has passed the active threshold can weaken in intensity, i.e., have an amplitude lower than the active amplitude threshold $\sigma$, without being considered to have terminated. This favors persistence over the onset of new events. An example is the 2003–2004 MJO in Fig. 4(b) where MJO weakens below the active threshold in Phase 3 and then reemerges after four phases, in Phase 8. We do not impose any constraints on the time that an event can stay below the active amplitude threshold before regaining strength as long as it stays above the termination threshold $r_{T}^{MJO}$ (Straub (2013) called events that evolved from weaker MJOs as “intensification” events). Here the termination threshold is $r_{T}^{MJO} = 0.3$, chosen to be slightly lower than the initiation threshold $r_{P}^{MJO} = 0.5$. When the MJO reaches the termination threshold, we call that event a terminal event. Using the approach presented here, every primary event will have a terminal event associated to it. Overall, higher initiation and termination thresholds discard events that do not complete half a cycle in the phase space despite reaching a significant amplitude. On the other hand, lower initiation and termination thresholds increase the length of the events and favor persistence and reemergence, but in the limit risk identifying a single event over the entire period. By favoring persistence, lower thresholds also decrease the number of
Fig. 6. Distribution of events per phase for varying (a) initiation (0.3 – 0.8), and (b) termination (0.1 – 0.6) threshold values. The termination threshold is always kept below the initiation threshold to increase persistence (with a constant difference \( r_{MJO}^{P} - r_{MJO}^{T} = 0.2 \)). We used the following combination for the initiation/termination threshold values \((r_{MJO}^{P}, r_{MJO}^{T})\): (0.3, 0.1), (0.4, 0.2), (0.5, 0.3), (0.6, 0.4), (0.7, 0.5), and (0.8, 0.6). The primary events are relatively equally distributed among the phases. The distribution of terminal events shifts first to the right (similar to moving to the next phase in the 2D phase space diagram) for the increase of the threshold in the range 0.2 – 0.5, and then shift back to the left when termination occurs at \( r_{T}^{MJO} = 0.6 \). Phase 7 is the most frequent for \( r_{T}^{MJO} = \{0.4, 0.5\} \), but the algorithm starts to miss significant MJO events above the value \( r_{T}^{MJO} = 0.3 \) (see Appendix B).
primary events, and generally increase the percentage of primary events that develop into full events.

We performed robustness tests for the termination threshold with values in the range $0.1 - 0.6$, and the resulting climatology of terminal events is shown in Fig. 6(b). The results are fairly robust for $r_{T}^{MJO} = \{0.2, 0.3\}$, but the 0.2 threshold starts to identify boreal summer events as candidates for full MJOs (see Appendix B for details). For low threshold values (e.g., $r_{T}^{MJO} = 0.1$) the algorithm starts to merge events from consecutive years. When $r_{T}^{MJO}$ is higher than our chosen 0.3 threshold the algorithm starts to miss MJO events which qualitatively appear to be significant (Appendix B). Interestingly, Phase 7 (Western Pacific) becomes the most frequent termination phase for $r_{T}^{MJO} = \{0.4, 0.5\}$, but these values suffer from poor identification of significant events. In the MJO index, 44% of the data samples have an amplitude greater than the initiation threshold 0.5, and 63% have an amplitude greater than the termination threshold 0.3.

Not all primary events develop into full MJO events. Over the 23 years of data there is a total of 36 primary events out of which only 27 go through at least four full phases in the phase space and are considered candidates for full MJOs. The 27 primary and their associated 27 terminal events are displayed in Fig. 7. Out of these 27 primary MJOs only 23 reach the amplitude threshold of one standard deviation and are considered full MJOs. Four primary events (1994, 1994, 2000, 2002) never attain the active amplitude threshold before terminating and therefore remain too weak to be considered active MJOs. Note that the choice of the active threshold value is ad hoc and could discard weak but valid MJO events. An alternative approach would be to use multiple thresholds to classify an MJO into different categories, e.g., strong, medium or weak. Overall, we identify roughly one full MJO event per year, but there is one year (1994) without a full MJO and two years (1989, 1991) with two full MJOs.

So far, we have discussed the question of when does the MJO initiate and terminate. In the following we will be looking at where does the MJO initiate and terminate? To answer this question we first need to estimate the statistics of primary and terminal events relative to the phases in Fig. 3(a) and determine the climatology of full MJO events. These statistics are summarized in Fig. 8 and Table 1. We point out that the precise values of the phase dependent occurrence frequencies should be interpreted with caution due to the low number of events detected per phase.

The primary and terminal events associated with full MJOs are relatively equidistributed among the phases, however we have observed some patterns that we detail in the following. In particular, the Indian Ocean (Phases 2 and 3 together) is the region with the highest frequency of occurrence of both primary and terminal events, though the difference in frequency with the other regions
Fig. 7. Time series of the NLSA-based MJO index amplitude $r_t^{\text{MJO}}$, showing a total of 23 full MJOs (red background). We identified a total of 36 primary and terminal events in the MJO index, but show here only the 27 events that go through at least four full phases in the phase space in Fig. 3(a) (the nine events that are too short, i.e., less than half a cycle, are not shown). The primary and terminal events are marked by blue and red $\star$, respectively. Out of these 27 events, 23 reach the active threshold of one standard deviation and are therefore considered full MJOs (red background). Four of them (blue background) never reach the active threshold of one standard deviation before terminating, and are considered not to mature into active MJOs. Initiation and termination occur when the amplitude of the index reaches the thresholds of $r_P^{\text{MJO}} = 0.5$ and $r_T^{\text{MJO}} = 0.3$, respectively.

<table>
<thead>
<tr>
<th>Event type</th>
<th>NLSA MJO phase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Primary (%)</td>
<td>9</td>
</tr>
<tr>
<td>Terminal (%)</td>
<td>17.5</td>
</tr>
<tr>
<td>Total (%)</td>
<td>13.25</td>
</tr>
<tr>
<td>Circumnavigating (%)</td>
<td>9.66</td>
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</tbody>
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Table 1. Statistics of the 23 full MJO events for the time period from 1984–2006 for each MJO phase as identified through the NLSA-based MJO index. Results are shown in percentages. Circumnavigating events are a subset of full events and do not count towards the total number of events per phase.
Fig. 8. Frequency of primary and terminal events for each MJO phase associated with the 23 full MJOs that occurred between years 1984–2006. The initiation $r_{P}^{\text{MJO}} = 0.5$ and termination $r_{T}^{\text{MJO}} = 0.3$ thresholds are denoted with blue and red circles, respectively. The individual primary and terminal events are denoted with blue and red $\star$.

is relatively small. During the 23-year observation period, a total of 7 primary events (3 events in Phase 2 and 4 events in Phase 3) and 8 terminal events (5 events in Phase 2 and 3 events in Phase 3) occurred in the Indian Ocean. This result is consistent with the observations, as the MJO’s main basin for onset is thought to be the Indian Ocean (Matthews, 2008; Yoneyama et al., 2013). The next most frequent initiation happens in the Western Pacific (Phases 6 and 7 together) with a total of 6 primary events (3 events in Phase 6 and 3 events in Phase 7). Frequent termination is associated with the Western Hemisphere and Africa (Phases 8 and 1 together) with a total of 7 terminal events (3 events in Phase 8 and 4 events in Phase 1). Among more localized domains that witness frequent events are Phase 7 with 4 terminal and 3 primary events, the western Maritime Continent (Phase 4) with 4 primary and 2 terminal events, and Africa (Phase 1) with 4 terminal and 2 primary events. Phase 7 also emerges as the most frequent on average for different values of the termination threshold. The phase space diagrams for primary and terminal events, together with their trajectories after initiation and before termination, are shown in Figs. 15 and 16 of Appendix A. The index often travels a few full phases between initiation/termination and the active MJO threshold (one standard deviation), with less rapid transitions...
than in the case of RMM. A summary of the initiation/termination statistics for the RMM index is presented in Sect. 4.2.

Out of the 23 full MJOs, 21 are circumnavigating events, i.e., they go through at least one full cycle in the phase space, and many of them (15) actually circumnavigate the globe two or three times between onset and decay. The two events that do not complete a full cycle are the December 1990 – January 1991 event, prior to the 1991 MJO, and the spring-summer MJO of year 2000. The latter occurred after a weak MJO earlier that year and is associated with the strong La Niña event of year 2000 (Shabbar and Yu, 2009).

3.2 Boreal summer intraseasonal oscillation

The two-dimensional representation of the BSISO through the corresponding pair of NLSA eigenfunctions (Fig. 1(c, d)) is plotted in Fig. 9(a). The phase space is split into eight phases and its associated composite life cycle is reconstructed in Fig. 9(b). Note the difference in the propagating patterns of the MJO and BSISO. While MJO has a dominant eastward propagating pattern (Fig. 3(b)), BSISO propagates northeastward towards the Indian and Asian monsoon regions (Fig. 9(b)).

Unlike the MJO, the BSISO eigenfunctions follow a counterclockwise rotation in the two-dimensional phase space diagram in Fig. 9(a), corresponding to a northeastward propagation in the spatial domain. Similarly to MJO, an event is considered a full BSISO if: 1) the index amplitude crosses at least once the threshold of one standard deviation $\sigma$ (here $\sigma = 0.94$), and 2) it propagates through at least four full phases, i.e., half a cycle, in the phase space. Primary and terminal BSISO events are defined similarly to MJOs, but using different threshold values. Figure 10 shows the PDF of the BSISO index amplitude which is skewed, though to a significantly lesser extent than the PDF of the MJO index amplitude. To adapt to this feature of the BSISO index we choose slightly higher threshold values for initiation and termination. Specifically, we set the primary event onset to $r_{P}^{\text{BSISO}} = 0.8$ and the terminal event decay threshold to $r_{T}^{\text{BSISO}} = 0.6$. We have again chosen the termination threshold slightly lower (using the same difference of 0.2 as for MJO) than the initiation threshold to counter for local small fluctuations in the time series.

We identified a total of 79 primary and terminal events in the BSISO index between years 1984–2006, out of which only 31 events went through at least half a cycle in the phase space. All of the latter 31 events (Fig. 11) develop into full BSISOS as they all reach the active amplitude threshold of one standard deviation $\sigma = 0.94$. The lower intermittency of BSISO compared to
Fig. 9. (a) Two-dimensional phase space diagram for the BSISO eigenfunctions from NLSA. The black circle denotes the constant amplitude of one standard deviation. (b) Spatiotemporal reconstructions of convective anomalies associated with each phase of the BSISO index as depicted in (a).
Initiation and Termination of ISOs in NLSA-based Indices

Fig. 10. Probability density function of the NLSA-based BSISO index amplitude $r^{\text{BSISO}}_t$. The red and black dashed vertical lines denote the threshold for initiation at $r^{\text{BSISO}}_P = 0.8$ and the threshold for active BSISOs at one standard deviation $\sigma = 0.94$. The PDF of the RMM index amplitude is also shown for comparison.

Fig. 11. Time series of the NLSA-based BSISO index amplitude $r^{\text{BSISO}}_t$, showing a total of 31 full BSISOs (red background). We identified a total of 79 primary and terminal events in the BSISO index, but show here only the 31 events that go through at least four full phases in the phase space in Fig. 9(a). All these 31 events will reach the active threshold of one standard deviation and therefore develop into full BSISOs. Initiation and termination occur when the index amplitude reaches the thresholds of $r^{\text{BSISO}}_P = 0.8$ and $r^{\text{BSISO}}_T = 0.6$, respectively. The onset and decay of the BSISOs is marked by primary (blue ⋄) and terminal (red ⋆) events.
MJO (and the higher number of primary and terminal BSISO events detected) can be a genuine property of BSISO due to fewer periods of quiescence and higher frequency content, or corruption due to NLSA (e.g., a slight mixing with other intraseasonal modes). Out of the 79 primary events, 48 events do not develop into full BSISOs either because they do not reach the active threshold value or do not complete a half cycle in the phase space. The strongest of these events occur in summer-autumn, e.g., 1986–1987, 1989–1990, 1990–1991, 1991–1992, 1998–1999, 2003–2004, and 2005–2006, and several weaker ones occur during boreal winter. A lower termination threshold \( r_{T}^{BSISO} < 0.6 \) would have allowed the strong summer-autumn events to be merged with previous full BSISOs. However, because the decay of the amplitude of the index is often very abrupt and approaches values close to zero before reemergence (e.g., the summer-autumn 1986–1987 event), the value of the threshold would have to be chosen very small, resulting in some BSISO events extending into boreal winter. The primary “BSISO” events that occur in boreal winter are generally weaker and do not develop into full BSISOs. These events may be a consequence of a mild amount of corruption in the NLSA eigenfunctions as stated above. Overall, the influence of the threshold values is stronger for BSISO compared to MJO.

The frequency of primary and terminal BSISO events per phase is displayed in Fig. 12 and Table 2. The statistics for the BSISO initiation and termination are less equally distributed compared to MJO. Specifically, the highest activity, both in terms of initiation and termination, is observed in the wet phase over the Indian Ocean (Phases 6 and 7 together) with a total of 23 events out of which 11 are primary events (7 events in Phase 6 and 4 events in Phase 7), and 12 are terminal events (5 events in Phase 6 and 7 events in Phase 7). Significant initiation also occurs in Phase 2 (the dry phase of BSISO over the Indian Ocean) with 10 primary events. Two localized domains that experience a high frequency of terminal events are Phase 8 with 7 events and Phase 4 with 5 events. Phase 8 can be associated with the final decay of the BSISO with no new event following, while Phase 4 can be associated with an event that terminates and is shortly followed by a new event, i.e., the start of a new dry phase in the Indian Ocean. S15 observed that the BSISO tends to initiate with a dry phase over the eastern Indian Ocean, equivalent to Phase 2 in Fig. 9(b), with 32% of events initiating in this phase in Fig. 12.
Fig. 12. Frequency of primary and terminal events for each BSISO phase associated with the 31 full BSISOs that occurred between years 1984–2006. The initiation $r^{\text{BSISO}}_P = 0.8$ and termination $r^{\text{BSISO}}_T = 0.6$ thresholds are denoted with blue and red circles, respectively. The individual primary and terminal events are denoted with blue and red ⋆.

<table>
<thead>
<tr>
<th>Event type</th>
<th>NLSA BSISO phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary (%)</td>
<td>13 32 6.5 3 3 23 13 6.5</td>
</tr>
<tr>
<td>Terminal (%)</td>
<td>3 6 10 16 3 16 23 23</td>
</tr>
<tr>
<td>Total (%)</td>
<td>8 19 8.25 9.5 3 19.5 18 14.75</td>
</tr>
</tbody>
</table>

Table 2. Statistics of the 31 full BSISO events for the time period from 1984–2006 for each BSISO phase as identified through the NLSA-based BSISO index. Results are shown in percentages.
Fig. 13. NLSA-based ISO bimodal index \((r^{MJO}_t, r^{BSISO}_t)\). Following Kikuchi et al. (2012), three periods are plotted in different colors: June–October (red), December–April (blue), and otherwise (gray). The solid lines indicate the threshold for significant ISO activity corresponding to one standard deviation of the MJO and BSISO index amplitudes. Observations are classified into three categories according to their associated regions as follows: I) active MJO states, III) active BSISO states, II) active MJO or BSISO states. The majority of significant active ISO states are either MJO states (I) occurring in December–April, or BSISO states (II) occurring in June–October.

4 Discussion

4.1 Comparison of MJO and BSISO statistics

Following Kikuchi et al. (2012) we build an ISO bimodal index (Fig. 13) using the amplitude of the individual MJO and BSISO indices \((r^{MJO}_t, r^{BSISO}_t)\). A point in this plane corresponds to significant instantaneous ISO activity if either the MJO or BSISO index amplitude is higher than or equal to one standard deviation. The NLSA-based bimodal index has high discriminating power, i.e., the majority of significant states are either purely active MJOs or active BSISOs. Moreover, the active MJO and BSISO states are correctly assigned to the boreal winter and boreal summer, respectively. S15 compared the nonlinear NLSA ISO bimodal index with a linear ISO bimodal index based on the singular spectrum analysis (SSA, (Ghil et al., 2002)), and observed a significantly higher classification power for NLSA compared to SSA. According to the analysis in Stachnik et al. (2015), the difference between the climatology of MJO events for year-round vs. boreal winter is only 1-2 %. In our case, such a comparison is not necessary since
the NLSA-based MJO and BSISO modes have strongly seasonal activity in the boreal winter and boreal summer, respectively.

The number of primary and terminal events and their distribution per phase depends on the threshold values used for initiation and termination. The active threshold value (here one standard deviation) has a direct impact on the number of full ISOs detected. More importantly, these values affect the when and where of each individual event’s initiation and termination. As the transitions between phases are smooth, even small changes in the threshold values will affect the initiation/termination phase assigned to each primary/terminal event. The index often travels through two or three phases between initiation (e.g., for MJO at $r_P^{MJO} = 0.5$), and the active threshold of one standard deviation. Thus, from a prediction standpoint, the choice of the initiation and termination threshold values has a strong influence on the identification of individual events.

### 4.2 Comparison with other indices

In previous works (Matthews, 2008; Straub, 2013; Stachnik et al., 2015) the climatology of primary, terminal, full, and circumnavigating MJO events has been analyzed through the lens of EOF-based indices of univariate fields or the multivariate RMM index (Wheeler and Hendon, 2004). RMM uses latitudinal averaging over the tropical belt 15°S–15°N and therefore captures mainly the zonal component of equatorially symmetric patterns, i.e., the eastward propagation characteristic of the MJO. The spatial patterns associated with the first principal components of the RMM index are reconstructed separately for boreal winter (December–January–February) and boreal summer (May–June). The weakening of the eastward-propagating signal during boreal summer allows the spatial reconstructions to recover the BSISO-specific northeastward-propagating pattern, however to a significantly lesser extent than possible through NLSA reconstructions.

The statistics of primary MJO events have been studied by Matthews (2008) and Straub (2013) using EOF-based indices. Stachnik et al. (2015) studied the statistics of both primary and terminal events, using also EOFs. Straub (2013) performed multiple analyses using only OLR, only wind, or both OLR and wind, and observed that the OLR contributes minimal information to the RMM index. The individual events identified using these different data sources differed significantly in their distribution over the phases, i.e., the same event initiated in different phases for the different data sources used. During the boreal winter (October–May) of the 32-year time interval (1979–2010), a total of 28 primary events was detected in the RMM index (both cloudiness and circulation) and
a slightly lower number of primary events in the cloudiness-only (27 events) or circulation-only (23 events) index. Stachnik et al. (2015) analyzed approximately the same time period (1979–2012) as Straub (2013). In their analysis, a total of 154 primary and terminal events were identified in the year-round RMM index and 91 primary and terminal events for the boreal winter (November–April). The discrepancy between the number of events identified via the two analyses seems to be a consequence of the details used in the implementation of the event selection algorithms. Straub (2013) used a length of seven days as a requirement for continuous eastward propagation at an average amplitude less than 1 prior to initiation as well as after initiation at an amplitude above 1. In Stachnik et al. (2015), primary events occur at the first days with an amplitude greater than unity, and terminal events either when the index amplitude suddenly drops below a lower buffer (RMM < 0.9) or when it drops below unity but stays above the lower buffer (RMM ≥ 0.9) for an extended period of time (more than three days). In our case, initiation occurs whenever the index amplitude stays below the initiation threshold for three consecutive days and crosses the threshold for the next two consecutive days. We used this requirement to alleviate the effects of a small amount of high frequency (diurnal) variability present in our MJO eigenfunctions. Overall, we obtained a frequency of events closer to the results presented in Straub (2013), that is, roughly one event per year.

A major difference between NLSA and EOF-based indices is intermittency and strong seasonality. In NLSA, the MJO and BSISO indices are naturally (i.e., without seasonal partitioning) active during the boreal winter and boreal summer, respectively. Moreover, during their respective periods of activity, the amplitudes of the NLSA indices exhibit significantly weaker high-frequency oscillations than RMM (see Fig. 14 ahead). As a result, the spatiotemporal MJO and BSISO patterns extracted by NLSA have a coherent evolution amplifying in a quasiperiodic manner on the boreal winter and boreal summer, respectively. In particular, NLSA captures the MJO as a propagating dipole of enhanced/suppressed activity in a wavetrain-like signal; that is, before the center of enhanced convection decays in the western Pacific, another center of enhanced convection is already present over Africa and the Indian Ocean (see Phase 1 in Fig. 3(b)).

The individual MJO events detected in other indices such as RMM correspond to one circumnavigation in NLSA. Most of the NLSA MJOs have 2–3 circumnavigations leading to the wavetrains depicted in the spatiotemporal reconstructions (Movie 1(f) of S15). RMM is active all year-round (Fig. 1(f)), and by construction of the PC time series it emphasizes the growth and decay of the static patterns in the corresponding EOFs. The number of events identified via NLSA is significantly smaller compared to RMM due to strong seasonality and
slowly-varying amplitude in the former, favoring multiple circumnavigations in one event.

The power spectra of the NLSA ISO and RMM index amplitudes are displayed in Fig. 14. Due to the removal of interannual variability and seasonal cycle together with its first three harmonics, RMM displays reduced spectral power on these timescales compared to NLSA. On the intraseasonal scale, the amplitude of the NLSA MJO index displays more power at lower frequencies (1/90-1/50 days), while RMM gives relatively equal weight to all intraseasonal frequencies with more power than NLSA MJO at higher frequencies (1/40-1/30 days). The amplitude of the NLSA BSISO index displays more power on the intraseasonal scale compared to both the NLSA MJO and RMM index amplitudes. Moreover, RMM displays more power at higher frequencies, i.e., below the intraseasonal scale, indicating a higher mixing between timescales compared to NLSA. We emphasize the ability of NLSA to extract relevant power on the intraseasonal scale comparable to RMM, despite no preprocessing of the data.

5 Conclusion

In this paper, we have presented a statistical analysis of intraseasonal oscillation events using recently proposed indices (Székely et al., 2015) for the boreal winter and boreal summer ISOs that are based on the nonlinear Laplacian spectral analysis (NLSA) method. Following previous studies (Matthews, 2008; Straub, 2013; Stachnik et al., 2015) we defined primary, terminal, full, and circumnavig-
gating events relative to these new indices. While ISO events have been largely studied through the lens of the year-round active RMM index, here we performed two separate analyses for the boreal winter and boreal summer ISOs. To our knowledge, this is the first study considering the climatology of the BSISO as a stand-alone phenomenon with its unique northeastward-propagating pattern. Its importance stems from its strong impact on the Indian and southasian monsoon’s onset and active/break phases (Goswami, 2011).

While RMM combines the convection and circulation components of the atmosphere into one index, the NLSA indices are extracted from pure convection data without preprocessing (in particular, without seasonal partitioning, bandpass filtering, or zonal averaging). Straub (2013) shows that the cloudiness component of the data contributes only little new information to the RMM index compared to the circulation component. Matthews (2008) also recovered MJO signals from pure convection data after removal of the seasonal cycle and its first three harmonics and after filtering using a 20–200-day filter to isolate the intraseasonal signal. In this paper, we showed that using only convection data and with no preprocessing we are able to objectively recover distinct families of NLSA eigenfunctions for the dominant boreal winter and boreal summer ISOs.

We found that 91% of the full MJOs identified for the period 1984–2006 were circumnavigating events with one or multiple circumnavigations. Also, the NLSA MJO index has very little to no retrograde (westward) propagation. Over the 23 years of observations we found 23 full MJO events corresponding roughly to one event per year, but there were years with zero or two events. Our definition of the primary and terminal events favored reemergence over the decay of an event and the onset of a new primary event. We imposed no constraints over the time that an ISO could stay below the active threshold value as long as it was not decaying below the termination threshold. For BSISO, we found a higher frequency of primary and terminal events with roughly 3.5 events per year, however only 40% of these events developed into full BSISOs (roughly 1.3 events per year).

Both MJO and BSISO exhibit strong activity in the Indian Ocean, with the centers of convection moving eastward and northeastward, respectively. The Indian Ocean has been identified in previous studies and observations to be the main basin for MJO initiation (Matthews, 2008; Yoneyama et al., 2013). Additionally, our results indicate that the Indian Ocean plays an important role in BSISO initiation, with 38.5% of primary events initiating in the dry phase over the Indian Ocean (Phases 2 and 3 together) and 36% in the wet phase over the Indian Ocean (Phases 6 and 7 together). The Indian Ocean also witnesses significant termination for both MJO and BSISO, with 35% of MJOs decaying in this region, and 39% of BSISOs terminating during the wet phase over the Indian
Ocean (Phases 6 and 7 together). Our results on MJO statistics are consistent with previous research (Matthews, 2008; Straub, 2013; Stachnik et al., 2015) in that the onset and decay can occur in any phase of the MJO phase space (see also Appendix A). However, to our knowledge no separate analysis of the boreal summer ISO via a stand-alone BSISO index has been performed previously.

An alternative scenario worth considering is to study ISO initiation conditional on a given region (e.g., Indian Ocean) with more relaxed constraints on the amplitude of the index. Straub (2013) also considered this scenario by stating that by the time the amplitude of the RMM index reaches the initiation threshold of one standard deviation, the MJO might in fact have already initiated. This question requires further analysis such as the existence of precursors for initiation/termination (similar to Stachnik et al. (2015)) other than just the magnitude of the MJO index, which we plan to pursue in future work. We are also currently investigating other data sources, such as upper- and lower-level zonal winds, to construct circulation-based NLSA indices and compare them with state-of-the-art circulation and convection-circulation indices.

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A Primary and terminal MJO events

In this appendix we plot the primary and terminal events identified in the NLSA-based MJO index together with their trajectories. For primary events we display the trajectories after initiation (at $r_{MJO}^{P} = 0.5$) for the ensuing 30 days. It is important to note that in most cases the transition of the index between phases happens slowly, with a few exceptions, e.g., the 1986-01-01, 1989-03-24 events, where the transition from the initiation threshold to one standard deviation occurs in the same phase. For terminal events we display the trajectories 30 days prior to termination. The transitions between phases from one standard deviation to termination at $r_{T}^{MJO} = 0.3$ are smooth, however slightly more rapid transitions are seen than at initiation. We emphasize that, due to the smooth transitions between phases, the choice of the threshold values for initiation and termination do play a role in the identification of the onset and decay of each event. As mentioned in Sect. 5, one possible scenario is that all ISOs initiate in (or near) the same region, such as the Indian Ocean, but at different intensities.
(amplitudes). Figure 17 shows the composites for primary and terminal events and their trajectories 30 days after initiation and 30 days prior to termination, respectively. While the transition speed of primary events is roughly equal among the phases, terminal events seem to transition faster when termination occurs in Phases 7, 8 and 1. We note that due to the very small number of samples, i.e., 1–4 events for each phase, it is difficult to accurately interpret these composites.

B Influence of threshold values on MJO event detection

In this appendix we provide details on the dependence of primary and terminal event detection on the corresponding threshold values. We always keep the termination threshold below the initiation threshold (with \( r_{MJO,P} - r_{MJO,T} = 0.2 \)) to increase persistence and handle noisy observations. As shown in Figs. 18 and 19 when \( r_{MJO,T} = 0.1 \) the algorithm merges events from consecutive years, for \( r_{MJO,T} = \{0.2, 0.3\} \) the results are fairly robust, and when \( r_{MJO,T} \) increases above 0.3 the algorithm starts to miss significant events, such as the 1997–1998 MJO event at \( r_{MJO,T} \geq 0.4 \) and the 2000–2001 event at \( r_{MJO,T} \geq 0.5 \). The events also get shorter as the threshold values increase. We also included Fig. 20 where we keep the initiation threshold at the value used throughout the paper (i.e., \( r_{MJO,P} = 0.5 \)), and vary only the termination threshold from \( r_{MJO,T} = 0.3 \) (used in the paper) to \( r_{MJO,T} = 0.4 \). By modifying only the termination threshold the algorithm misses the 1998–1999 full MJO event.

References

Fig. 15. Primary events associated with full MJOs and their trajectory for the ensuing 30 days. The trajectories often show a slow increase in amplitude between initiation at $r_{P}^{\text{MJO}} = 0.5$ and one standard deviation. The index often travels between one and three full phases between these two values.
Fig. 16. Terminal events associated with full MJOs and their trajectory 30 days in advance. The trajectories often show a slow decrease in amplitude between one standard deviation and termination at $r_{MJO}^T = 0.3$. The index often travels between one and three full phases between these two values.
Fig. 17. Per phase composites for the primary and terminal events from Figs. 15 and 16, and their trajectories 30 days after initiation and 30 days prior to termination.
Fig. 18. Time series of the NLSA-based MJO index amplitude $r_M^{MJO}$ for different values of the initiation and termination thresholds. The termination threshold is always kept lower than the initiation threshold, i.e. 0.2 difference.
Fig. 19. Time series of the NLSA-based MJO index amplitude $r_{MJO}$ for different values of the initiation and termination thresholds. The termination threshold is always kept lower than the initiation threshold, i.e. 0.2 difference.

Fig. 20. Time series of the NLSA-based MJO index amplitude $r_{MJO}$ for initiation at $r_{MJO} = 0.5$ and termination at $r_{MJO} = 0.4$. 

(a) Initiation 0.7, termination 0.5

(b) Initiation 0.8, termination 0.6


