Surface Stress Induced Entrainment in Stratified Fluids

Shilpa Khatri
Roberto Camassa
Kevin Mcgoff
Candace Gilet
Department of Mathematics
University of North Carolina at Chapel Hill
Chapel Hill, NC 27599

2003

1 Introduction

The ocean and other large bodies of water have densities that vary with depth. The stratification within these bodies of water is due to flow of different density fluids. The different densities are produced from variations in salinity and temperature. Gravity causes salt to move towards the bottom and the sun heats the fluid closer to the surface, creating a continuous stratification in the water. Layers are formed when there is a step in the density over a short depth of water. Layers can be created by bulk fluid flow, when a body of water with some density is moved above water with another density. Another way layers are formed is through localized stirring of the fluid. Stirring, turbulent fluid flow, of a continuously stratified body of water homogenizes the fluid to the average density. If these turbulent motions are contained to a region locally, then only the fluid in that region is homogenized, and there is a noticeable step in density between this area and the surrounding fluid. For example, on the surface of the ocean, wind stress creates stirring from the surface to a certain depth in the water; the forces from the turbulent flow homogenize the fluid being stirred and create two layers separated by a density jump [1].

The transport of fluid and particles between the two layers is due to turbulent motion and diffusion. Turbulent motions are difficult to analyze because a first principle theory is nonexistent [4]. Therefore, to gain understanding of such motions, we must study experimental evidence quantifying the properties and parameters.

In this thesis, we observe the mixing between two layers of different densities. These density layers are due to differences in salinity. Mixing between layers occurs when there is entrainment, transport of lower layer fluid into the upper layer, in the body of water. This entrainment occurs at the interface between the upper and lower layers. This experiment is specifically looking at surface stresses emulating wind forces, which are applied using horizontal jets on the free surface of two layers of stratified fluid. These stresses generate turbulent motion in the upper layer while the bottom layer continues to have no turbulent motion. Due to these forces, salt is entrained from the bottom layer into the upper layer, thereby increasing the salinity of the upper layer. The stirring in the upper layer is much quicker than the entrainment. Therefore, the motion in the upper layer creates enough flow, compared to the amount of salt that is entrained, to homogenize this top layer, while the bottom layer remains homogeneous. Even though the upper layer now contains more salt and has a higher density, approximately two layers remain, just as we began with [2].

Since these experiments are conducted on a much smaller scale compared to the ocean it remains to be seen how these results can be scaled up to the situation in the ocean. Specifically, we are looking to have an understanding of the mixing rate, which, given the two layer assumption, is the rate at which the upper layer deepens. The main difference with respect to the ocean is the rigid boundaries present in the experiment. Two different configurations of the tank are used, each with different boundary conditions, allowing us to investigate the influence of these boundaries.

A theoretical model based on mass conservation and energy balances is created to analyze and understand the data collected. Using one experiment we find the empirical constant, k, and then we use this to deduce how well the theoretical model compares to the data from all the other experiments. Different values for the parameters (the velocity of the jets, the density of the bottom layer, and the height of the layers) are used to emulate different situations present in the ocean. Data are gathered to observe how much mixing occurs over time. The data are not collected point wise in the tank but rather as averages spatially.

2 Experiment

2.1 Apparatus

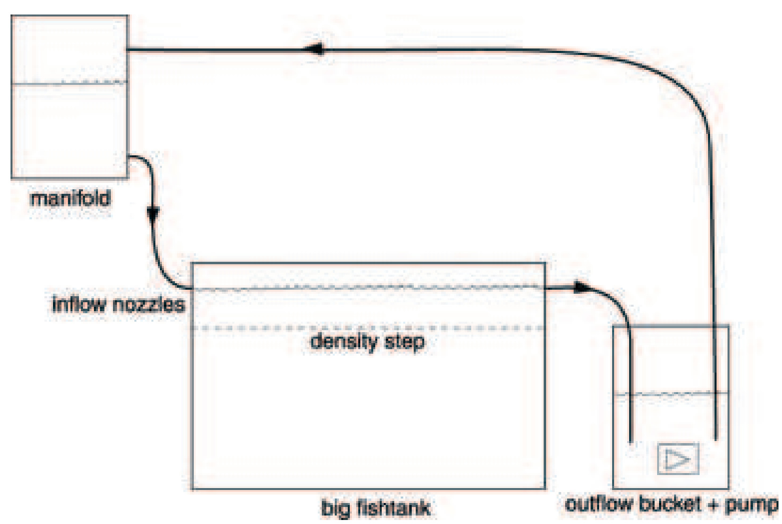

Figure 1: *This schematic shows the setup of the experiments.*

The experiment uses a 55.8 cm $\times$ 72.6 cm $\times$ 80 cm tank built out of 11/16 in. thick plexiglass. The design of the tank allows for two different configurations of the experiment. The two long sides of the tank have nine, 1/4 in. outer diameter stainless steel pipes built into the plexiglass. These pipes are 10 cm from the top of the tank, horizontally equally spaced at a distance of 2 and 7/8 in., allowing nine jets to come into the tank. The two end jets are placed at 1 and 7/16 in. (half the inter-jet distance) from the lateral walls. With this layout we try to minimize boundary effects on the flow induced by the end jets, by making the lateral walls act as image jets placed the same distance away from the end jets, as the neighboring jets. The pair of (shorter) opposite sides has seven, 1/4 in. stainless steel pipes, placed in the same fashion, allowing for seven jets to come into the tank. While one configuration is being used, the other pipes are corked shut.

The speed of the jets is controlled by a gravity feed from a cylindrical barrel, a manifold, placed above the tank. By making sure that the manifold maintains a sufficient volume of fluid at all times, we strive to keep the pressure head on each jet constant and independent of the flow fluctuations occurring in the flow upstream of the manifold. The barrel has nine equally spaced holes around the bottom, from where the feed lines to the jets depart. These lines are 238 cm long, 1/2 in. inner diameter flexible vinyl tubing and are connected to the stainless steel pipes on the tank. When the seven jets configuration is used, two of the jets run from the manifold to the outflow bucket, and they are placed at the same height as the jets flowing into the tank. The jets are maintained at a specific velocity by keeping the water level in the manifold constant. While one side of the tank is used for inflow, the opposite is used for outflow. The same 1/4 in. inner diameter, 157 cm long vinyl tubing is attached to the stainless steel pipes on this side. They run from the tank to the outflow bucket. This bucket sits on the floor next to the tank. A fishtank pump in the bucket and 3/4 in. inner diameter hose are used to create recirculation in the system, by pumping outflow back into the manifold (see figure 1).

2.2 Stratification

Figure 2: *The diffuser allows the freshwater to be poured slowly so that a sharp interface is created between the two layers.*

Each experiment requires the making of a controlled initial condition for the stratification in the tank; throughout this work we choose one that mimics a situation commonly observed in the ocean. The water in the tank is stably stratified by creating two homogeneous layers with a sharp interface between them. Creating this stratification profile requires some careful planning. First, the saltwater is prepared to the right density and is poured into the tank to a specified height. The saltwater is prepared by mixing in Diamond Crystal Extra Coarse Solar salt with freshwater. The amount of salt necessary to produce a particular density is done using a fit of salinity versus density,

$$\rho = \rho_f + \alpha_T (T - 25) + 0.0063S, \qquad (1)$$

where ρ is the desired density, ρ_f is the density of the freshwater, and α is the average thermal coefficient, $3.22 * \times 10^{-4} g/cc/℃$ [6]. (The density of the freshwater used in the experiments shown here ($\rho_f = 0.995$) has come into question as of summer 2003 due to experimental error. Since then, a freshwater density of 0.997 has been used in experiments conducted in the lab.) The amount of salt needed to produce a specified salinity is found using a fit created by Dr. Richard McLaughlin and Ryan McCabe [7],

$$Amount\ of\ salt = \frac{desired\ salinity \times mass\ of\ water}{1000}. \tag{2}$$

Once the salt is placed in the water, the Eastern Mixers model E-2T mixer is used to dissolve the salt. To prevent a temperature difference from influencing the experiment, we allow the freshwater and saltwater to sit in the room for a day or two before creating the stratified fluid. Even then, we usually maintain a degree or two difference between the two layers, with the saltwater being cooler. This resulting temperature difference is probably due to the fact that there is less freshwater sitting in bulk than saltwater.

A diffuser, made of Styrofoam, sponge, and pvc piping, is used to pour the freshwater layer on top of the saltwater (see figure 2). This diffuser allows the flow to be slow enough so that mixing at the interface between layers is very small, usually in the range of 3.58 cm. To be able to visualize the difference between the two layers when conducting the experiment and in a video, the bottom (saltwater) layer is dyed using food coloring.

2.3 Taking a Profile of the Tank's Stratification

Before the two layers are poured into the tank the density and temperature of both are measured. The density is measured, both by weighing a known volume and with the Orion conductivity probe, and the temperature is also measured with the probe. Once the stratified layers are in the tank, profiles of salinity and temperature are taken. These profiles give information of how salinity and temperature change with respect to height of the fluid, allowing the size of the mixed layer to be measured. The appendix explains some of the anomalies observed in the profiles measured.

Profiles are taken by dialing a calibrated Velmex slider attached with the conductivity probe through the tank. The same density fit as before, equation (1), is used to calculate density from the salinity measurements [6]. This fit was obtained using a digital scale accurate to .01 g, and a 2000 mL flask accurate to 0.50 mL. The probe was subsequently calibrated with respect to this fit [3].

After the layers are created, a density bead is placed into the tank to visualize the stratification. These spherical glass beads have a mass of 0.06 grams and a radius of 0.25 cm. In experiments in which the saltwater layer has a density of 1.01 g/mL, the bead used has a density of 1.005 g/mL, and when the bottom layer has a density of 1.02 g/mL, the bead has a density of 1.01 g/mL [3].

2.4 Running the Experiment

Once the salinity and temperature profiles of the layers are taken, the experiment is ready to run. The probe is now placed in the outflow bucket to measure the salinity and temperature of the outflow continuously in time. The pump is also placed in the outflow bucket to allow the outflow to be pumped back into the manifold, keeping the velocity of the jets constant. The manifold is filled with freshwater to the height which will maintain the velocity necessary for the specific experiment, and 13500 mL of freshwater are placed in the outflow bucket to allow recirculation to begin right away. The inflow jets, which were originally raised to prevent flow through them, are dropped and the pump is turned on to begin the experiment. The duration the jets run depends upon the parameters of the experiment. The experiment is stopped before the tank becomes completely homogeneous. After stopping the jets, another profile of the tank is taken in the same manner as before. The experiment is recorded throughout its duration with a Canon 3CCD GL1 Video Digital Camcorder. Besides keeping a record of the elapsed time and of the various parameters, the tape keeps a visual record of the mixing allowed by the initial dying of the lower, denser layer.

3 Two Configurations for Surface Forcing

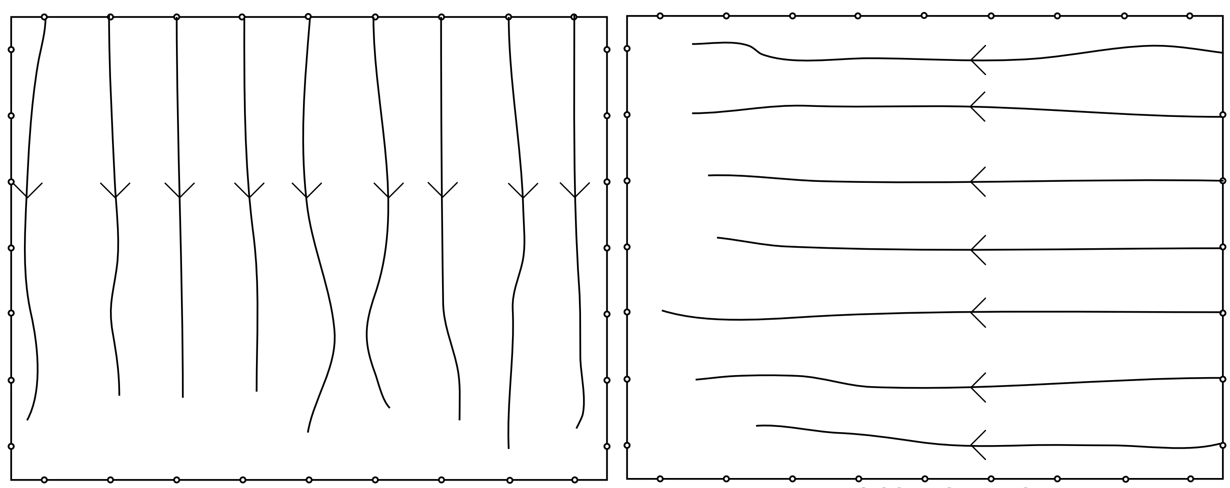

Figure 3: *The top view of the two configurations possible in the tank.*

Two setups are possible using our tank (see figure 3). By using two different configurations we are able to change the aspect ratio of the surface stress applied, and therefore test if different ratios produce different results in the mixing between layers.

Most of the experiments reported in this thesis have been done using the nine jets configuration. For this configuration, two different jet velocities are chosen by using two heights for the water level in the manifold. Based on these heights, the height of the water level in the manifold for the seven jets configuration needs to be calculated, if the total momentum supplied at intake is to remain constant. The other option, which we use in the two seven jets configuration experiments shown here, is to keep the same heights for this setup as the nine jets setup, maintaining the same kinetic energy and momentum per jet at intake.

3.1 Momentum Constant

A height is chosen in the manifold for the nine jets configuration and a height needs to be found for the seven jets configuration if the total momentum supplied to the tank is to remain constant for the two configurations. We begin by calculating the momentum each jet supplies to the tank, $\rho v_N^2 A$, where ρ is the density, v_N is the velocity of the jet when in a configuration with N jets, and A is the cross sectional area of the jet (measured internally in the pipes). Therefore, the total momentum entering into the tank per unit time when N jets are used in the configuration is,

$$\dot{P}_T(N) = \rho v_N^2 A N \tag{3}$$

Energy conservation is used to solve for v_N:

$$\frac{1}{2}\rho v_N^2 + \alpha v_N^2 = \rho g h_N \tag{4}$$

$$v_N = \left(\frac{\rho g h_N}{\frac{1}{2}\rho + \alpha}\right)^{\frac{1}{2}} . \tag{5}$$

Here, αv_N^2 is the energy lost due to friction and other dissipative forces, g is the gravitational acceleration, and h_N is the height from the jets to the surface of the water in the manifold for the N jets configuration. In the expression for the energy lost, α is the constant that scales the rate of work done by frictional forces.

Using equation (5) to substitute for v_N in equation (3),

$$\dot{P}_T(N) = \rho \frac{h_N \rho g}{\frac{1}{2}\rho + \alpha} A N . \tag{6}$$

Knowing h_N allows for the height of a M jets configuration to be found when momentum of the jets coming into the tank remains constant,

$$h_M = \frac{N}{M} h_N . \tag{7}$$

4 Flux and Velocity of the Jets

The flux of the jets is the volume of water that passes through the jets at the point at which they enter the tank over a time duration. For both the nine and seven jets configurations,

Figure 4: *From this image we can see a parabola fit to the flow from the jets.*

two fluxes and therefore velocities are used for the experiments presented here. These fluxes and velocities are maintained constant by keeping the water level constant in the manifold. For the nine jets setup this height is chosen to be 13.97 cm from the bottom of the manifold for one flux and 24.6 cm for the other flux. The heights are then calculated for the seven jets configuration if the momentum coming into the tank is to remain constant. Here, the two experiments run using seven jets use the same heights in the manifold as the nine jets experiments.

The flux is measured in two different ways, one directly and one indirectly. The obvious way to measure the flux of the jets is with a beaker and a stopwatch. With the mass of the water collected over a certain amount of time and the density of water, the flux is found in units of volume per time. The indirect method for measuring flux is to take an image of a jet and fit a parabola to this image by picking a few points on the jet (see figure 4). In this method we are assuming that the jet acts like a free falling object. Therefore, the velocity in distance per time can be calculated,

$$u = \left(\frac{g}{2a}\right)^{0.5}, \tag{8}$$

where u is the initial velocity coming into the tank, g is the gravitational acceleration, $980\frac{cm}{s^2}$, and a is the coefficient of the quadratic term in the parabola fit in units of $1/cm$. The flux can be calculated from the velocity using the cross sectional area of the jets and assuming the velocity is an average across this area. In the same manner, the velocity can be calculated from the direct measurement of the flux using this area. Flux and velocity are measured and calculated for both the nine and seven jets configurations with the two different heights in the manifold.

The cross sectional area of the jets are measured at the point at which they enter the tank with a caliper which is accurate to 0.02 cm. This value is used in the conversions from flux to velocity and vice versa. The average of the diameter of a few pipes was originally measured to be 0.4623 cm, and this value was used while working with the first model (section 6.1). While developing the model further, the average was measured again and this time found to be 0.4323 cm, which is the value used in the remaining work shown here.

5 Experiments Conducted

No. of jets	Density of Lower Layer (g/mL)	Velocity of Jets	Height of Freshwater Layer (cm)	Height of Saltwater Layer (cm)
9	1.01	higher	7	63
9	1.01	lower	7	63
9	1.01	higher	21	49
9	1.01	lower	21	49
9	1.015	higher	21	49
9	1.02	higher	21	49
9	1.02	higher	7	63
7	1.01	higher	7	63
7	1.01	lower	7	63

Table 1: *The nine different experiments we conducted.*

The apparatus used allows for a number of the parameters to be changed in the experiments. We ran nine experiments using both configurations. The different parameters which were varied include: the density of the saltwater (lower) layer, the velocity of the jets, and the heights of the two layers. The experiments we chose to run can be seen in Table 1.

To produce the two different velocities, we chose two different heights of water in the manifold. For the two experiments conducted using seven jets, we maintained the kinetic energy and momentum per jet constant to the nine jets configuration, by keeping the water level in the manifold the same as they had been for both velocities in the nine jets experiments.

We chose to vary these specific parameters for a number of reasons. Changing the velocity of the jets allows us to see the effect that varying forces have on the mixing. Also, changing the velocities is important because they were arbitrarily set based on the height of our manifold and the amount of water it could hold. Having flexibility with this parameter allows us to see the importance the velocity plays in the mixing.

We worked with two different heights of the freshwater layer because we believe the jets have a greater effect on the entrainment when the jets are closer to the interface. Varying the depth of the layer allows us to test this hypothesis. From our data, we observe that it did indeed take longer for the lower layer to be entrained into the top layer when the interface was farther away from the jets initially.

Finally, we believe a higher density in the lower layer causes the mixing to slow down when the velocity of the jets remains constant, due to the fact that more energy is needed to lift a greater mass into the upper layer. After using a lower layer with a density of 1.015 g/mL and seeing obvious differences from the 1.01 g/mL lower layer, we decided to run some experiments with a lower layer density of 1.02 g/mL. As expected, we observe that the mixing did slow down with an increase in density in the lower layer.

With the nine and seven jets configurations, we are using two different aspect ratios for the experiments, therefore having two different sets of boundary conditions. Two boundary conditions allows us to make sure that not just one configuration created the basic results seen with these different parameters.

6 Theoretical Models

6.1 Simplified Model

By making an assumption on the stratification profile as a function of time it is possible to find an expression for the potential energy that depends solely on the depth of the mixed layer. Possibly the simplest assumption is that the stratification maintains a two-layer structure throughout the experiment. This assumption is born out of the data, to various degrees of accuracy depending on the configuration. The underlying physical assumption is that the mixing by the stirring motion in the upper layer occurs on a time scale which is short enough compared with that of entrainment of the dense lower layer fluid across the interface region. Thus, as parcels of denser fluid move across the interface, they quickly get mixed into the upper layer. Accordingly, we should expect this assumption to agree better with the data for larger jet velocities (leading to more vigorous stirring) and larger density jump steps (slower transport across the interface due to the increased potential energy penalty).

Under this assumption, the conservation of mass for the systems is expressed by:

$$A\rho(t)\big(H - h(t)\big) + A\rho_l\, h(t) = M_T, \tag{9}$$

where $\rho(t)$ is the density of the upper layer at any time t, $h(t)$ is thickness of the lower layer of density ρ_l, H is the total height of the fluid in the experiment, and A is the area

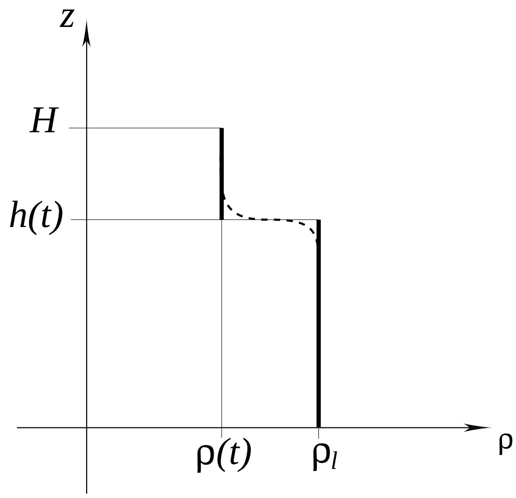

Figure 5: *The two-layer idealized configuration, with the upper layer of thickness $H - h(t)$ and density $\rho(t)$ eroding the lower layer of fixed density ρ_l. The dashed line sketches the profile commonly observed throughout the experiment.*

of the base of our tank. See figure 5 for a schematic of the setup. We are neglecting in this formula the amount of upper layer fluid contained in the inflow manifold and outflow bucket, respectively; the formula can easily be adjusted to take these two masses of fluid into account. Thus, the time evolution of the density is linked to the thickness $h(t)$ of the lower layer by the equation,

$$\rho(t) = \rho_l + \frac{H - h_0}{H - h(t)}(\rho_0 - \rho_l) \equiv \rho_l - \frac{\Delta h_0 \Delta \rho_0}{H - h(t)}\,, \tag{10}$$

where $\Delta h_0 \equiv H - h_0$ and $\Delta \rho_0 \equiv \rho_l - \rho_0$ are the initial thickness of the upper layer and the initial density jump between layers, respectively.

Similarly, for a two-layer fluid the total potential energy at any time t is

$$E_P(t) = gA\left(\int_0^{h(t)} \rho_l z\,dz + \int_{h(t)}^{H} \rho(t) z\,dz\right)\,, \tag{11}$$

with g the gravitational acceleration, and so

$$E_P(t) = \frac{gA}{2}\left(\rho_l (h(t))^2 + \rho(t)\big(H^2 - (h(t))^2\big)\right)\,. \tag{12}$$

Replacing $\rho(t)$ with expression (10) yields

$$E_P(t) = \frac{gA}{2}\left(\rho_l H^2 + (\rho_0 - \rho_l)(H - h_0)(H + h(t))\right)\,, \tag{13}$$

or, using the short-hand notation for the initial density jump and upper layer thickness,

$$E_P(t) = \frac{gA}{2}\left(\rho_l H^2 - \Delta\rho_0 \Delta h_0 (H + h(t))\right)\,. \tag{14}$$

The rate of change of potential energy is due to the rate of work done by the stirring fluid. In our case, this is the fraction of the work rate supplied by the pump that does not convert into kinetic energy in the fluid in the tank, potential energy increasing the height of the free upper layer surface, or heat produced in the tubes and the tank. In order to maintain a constant flux of fluid throughout the experiment, the rate of work done by the pump has to equal the flux of kinetic energy at any point in the system, and in particular it has to equal the kinetic energy flux supplied at the inflow jets. A small fraction of the energy from the pump might also be converted into wave motion, both at the free surface and at the intermediate layer, but this should be negligible with respect to the other forms of energy.

The kinetic energy flux from the jets is

$$J_E = \frac{1}{2}\left(\rho(t) u_*^2\right) N a\, u_* \,, \tag{15}$$

where u_* is the velocity of the fluid through one jet averaged across the jet's cross section, N is the number of jets, and a is the cross sectional area of one jet.

If we denote by k the fraction of the work rate that supplies potential energy, the potential energy balance can be written as

$$\frac{dE_P}{dt} = k J_E \,. \tag{16}$$

We can expect k to be dependent on several configuration parameters. Moreover, we should expect it to be a (possibly mildly) decreasing function of the thickness of the upper layer $H - h(t)$, since this is the distance between the jet location at the upper layer free surface and the interface, and as this distance increases the induced entraining motion in the vicinity of the interface must become (progressively) weaker. On the other hand, this can be compensated by the progressively lower density jump penalty.

The energy balance law (16) and the expressions for the potential energy (13) and density (10) determine the function $h(t)$ through the equation

$$\frac{dh}{dt} = -\frac{N a u_*^3}{g A}\, k \left(\frac{\rho_l}{\Delta\rho_0 \Delta h_0} - \frac{1}{H - h(t)}\right) . \tag{17}$$

The first term in this expression is clearly dominant, since the ratio $\rho_l/\Delta\rho_0 \gg 1$. Thus, to the leading order term, the solution to this ordinary differential equation (ODE), assuming k constant, is simply

$$h(t) = h_0 - C\, k\, t \,, \tag{18}$$

with the constant C defined by

$$C \equiv \frac{N a u_*^3}{g A} \frac{\rho_l}{\Delta\rho_0 \Delta h_0} \,,$$

that is, the lower layer erodes linearly in time. Of course, we cannot expect this law to hold indefinitely in time for our experiment. When the lower layer is fully eroded and the fluid mixed to its average density,

$$\rho_{av} = \rho_l - \frac{\Delta\rho_0 \Delta h_0}{H},$$

the potential energy has reached the maximum value allowed by the experiment and no longer increases.

6.2 Fitting and Comparing the Simplified Model to Data

We begin by trying to fit $h(t)$ found from only the dominant term of differential equation (18) to one set of our data to help us generate a k value. Since we are assuming k constant in this model, we can use this k value for all other data sets.

Since the density, $\rho(t)$, can be solved for once the height, $h(t)$, is known, we use the outflow density data of the nine jets experiment with a thicker freshwater layer, a higher velocity, and a lower layer density of about 1.015 g/mL to generate a value for k. We find k to be 0.0023 by fitting the model to the data (see figure 6). This value of k is found by initially arbitrarily choosing a k and then changing it as needed, to visually get the plot of the model to fit the data.

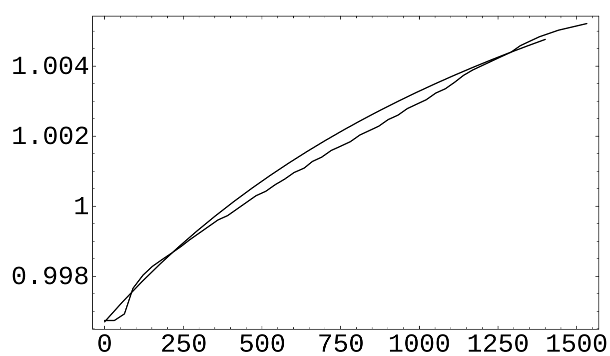

Figure 6: *By fitting the dominant term of the initial model to one experiment's data, we are able to find k equal to 0.0023*

By plotting the model with other data (see figure 7), it is obvious that this simplified model needs to be modified. Since we only use the dominant term of the differential equation, we decide to first try using the entire equation (17). Once again we find k by fitting the equation to that one experiment (see figure 8). This time we find k to be 0.000051. Once again we plot the model with this k value with all the other data sets (see figure 11, dashed line), and realize the model still needs more modification.

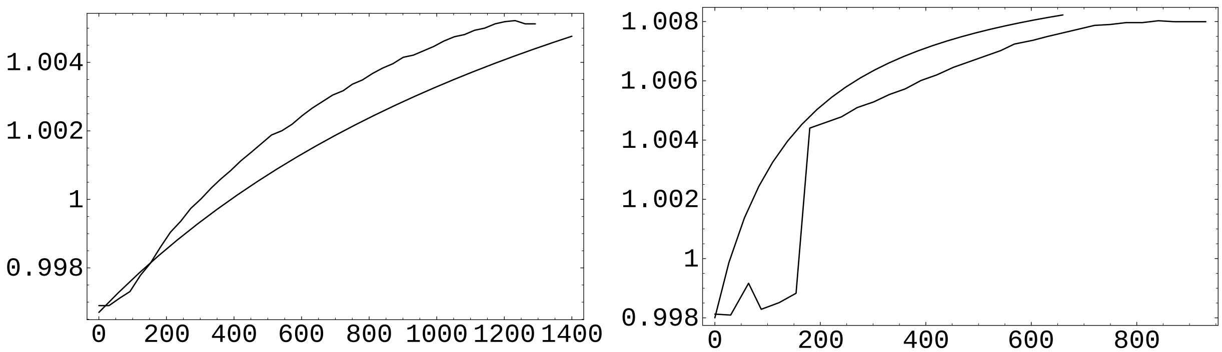

Figure 7: *Using k equal to 0.0023, we compared the model to other experiments.*

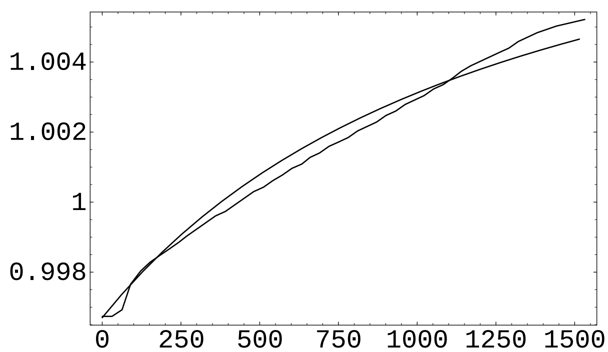

Figure 8: *By fitting the entire simplified model to one experiment's data, we are able to find a k value of 0.000051*

6.3 Improved Model

From the comparison of the simplified model to the data we are able to see that improvements upon this model are needed. We decide to improve by not neglecting the fluid in the manifold and the outflow bucket. Therefore, the conservation of mass for the system is then expressed by:

$$A\rho(t)\big(H - h(t)\big) + A\rho_l\, h(t) + \rho_b(t)V_b + \rho_m(t)V_m = M_T\,, \tag{19}$$

where $\rho_b(t)$ is the density of the fluid in the outflow bucket, V_b is the volume of the fluid in the outflow bucket, $\rho_m(t)$ is the density of the fluid in the manifold, and V_m is the volume of the fluid in the manifold. We know that the density of the fluid in the manifold and the outflow bucket changes in proportion to the total flux, f, of all the jets such that

$$\frac{d\rho_b}{dt} = \frac{f}{V_b}(\rho(t) - \rho_b(t)) \tag{20}$$

$$\frac{d\rho_m}{dt} = \frac{f}{V_m}(\rho_b(t) - \rho_m(t))\,. \tag{21}$$

From the conservation of mass, we can solve for $\rho(t)$,

$$\rho(t) = \frac{M_T - A\rho_l h(t) - \rho_b(t)V_b - \rho_m(t)V_m}{A(H - h(t))}. \tag{22}$$

From this equation, we calculated $\frac{d\rho}{dt}$ to be

$$\frac{d\rho}{dt} = \frac{(\rho(t) - \rho_l)}{(H - h(t))}\frac{dh}{dt} + \frac{f(\rho_m - \rho(t))}{A(H - h(t))}. \tag{23}$$

Even though we are now including the fluid in the manifold and outflow bucket, the potential energy of the fluid in the tank and the potential energy balance have the same expressions as before,

$$E_P(t) = \frac{gA}{2}\left(\rho_l(h(t))^2 + \rho(t)\left(H^2 - (h(t))^2\right)\right) \tag{24}$$

$$\frac{dE_p}{dt} = \frac{1}{2}k\left(\rho(t)u_*^2\right)N a\, u_*. \tag{25}$$

We can calculate the change in potential energy in the tank over time from equation (24),

$$\frac{dE_p}{dt} = \frac{gA}{2}(H^2 - h(t)^2)\frac{A(\rho(t) - \rho_l)\frac{dh}{dt} + f(\rho_w - \rho(t))}{A(H - h(t))} + gA(\rho_l - \rho(t))h(t)\frac{dh}{dt}. \tag{26}$$

Using this equation (26), the potential energy balance (25), and the expressions for $\rho(t)$ and $\dot{\rho}(t)$ (equations (22) and (23)), we can isolate $\dot{h}(t)$,

$$\frac{dh}{dt} = \frac{-\left(f(H + h(t))(\rho(t) - \rho_m) + \frac{1}{g}k\rho(t)u_*^3 aN\right)}{A(\rho_l - \rho(t))(H - h(t))}. \tag{27}$$

The equations for $\dot{h}(t)$, $\dot{\rho}_m(t)$, $\dot{\rho}_b(t)$, and $\rho(t)$ form a set of differential equations which we use to model the data.

6.4 Fitting and Comparing the Improved Model to Data

We are not completely finished working with this model but have begun by working with one experiment and trying to find a k value once again. In this improved model, we begin with the assumption that k is constant. To come up with this k value we try to fit $\rho_b(t)$ to the outflow data. By fitting the model to the experiment with a lower layer density of 1.01g/mL, a upper layer height of 21 cm, and using jets with the lower speed, we found $k = 0.0040$ (see figure 9). We have started looking at how this model fits with some of the other data but have not really worked to refine the k value, by possibly making it a function of height or density (see figure 11, solid line).

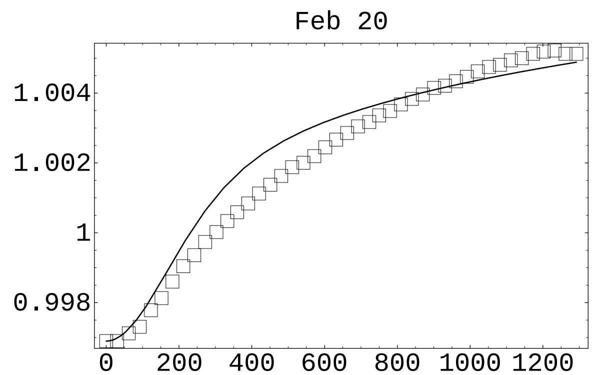

Figure 9: *We fit the model to this one experiment to find a k value of* 0.0040

6.5 Comparison of the Improved and Simplified Models

Since the only difference between the two theoretical models presented here is that the improved model takes into account the water in the manifold and the outflow bucket, we are able to obtain the same results if we let the water in the bucket and manifold approach zero. We did this by setting V_b and V_m to 0.01 mL in the modified model and comparing the results to the simplified model, while all other constants, including k, remained the same (see figure 10). From this figure we observe that in the limit of vanishing extra fluid, the two models become equivalent.

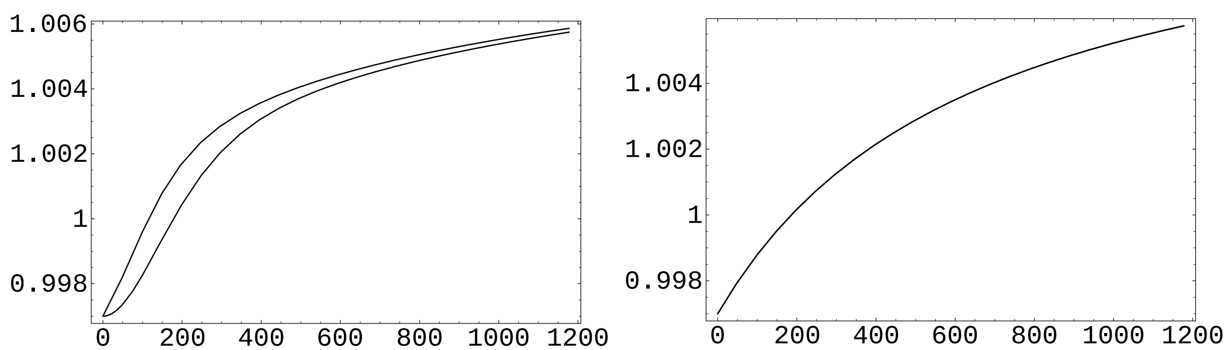

Figure 10: *The comparison on the left shows the two models when the water in the manifold and outflow bucket is included in the modified version, while the figure on the right shows the two models when that amount is approaching zero.* ($\rho_{saltwater} = 1.01g/mL$, $h_{freshwater} = 21cm$, *and lower velocity) (The values of* k, *the cross sectional area of the jets, and the flux measurements used in this comparison are not the values that were finally used in the models shown in this thesis. This figure still makes the point that if the constants are equal and the external fluid approaches zero, the two models are equivalent.)*

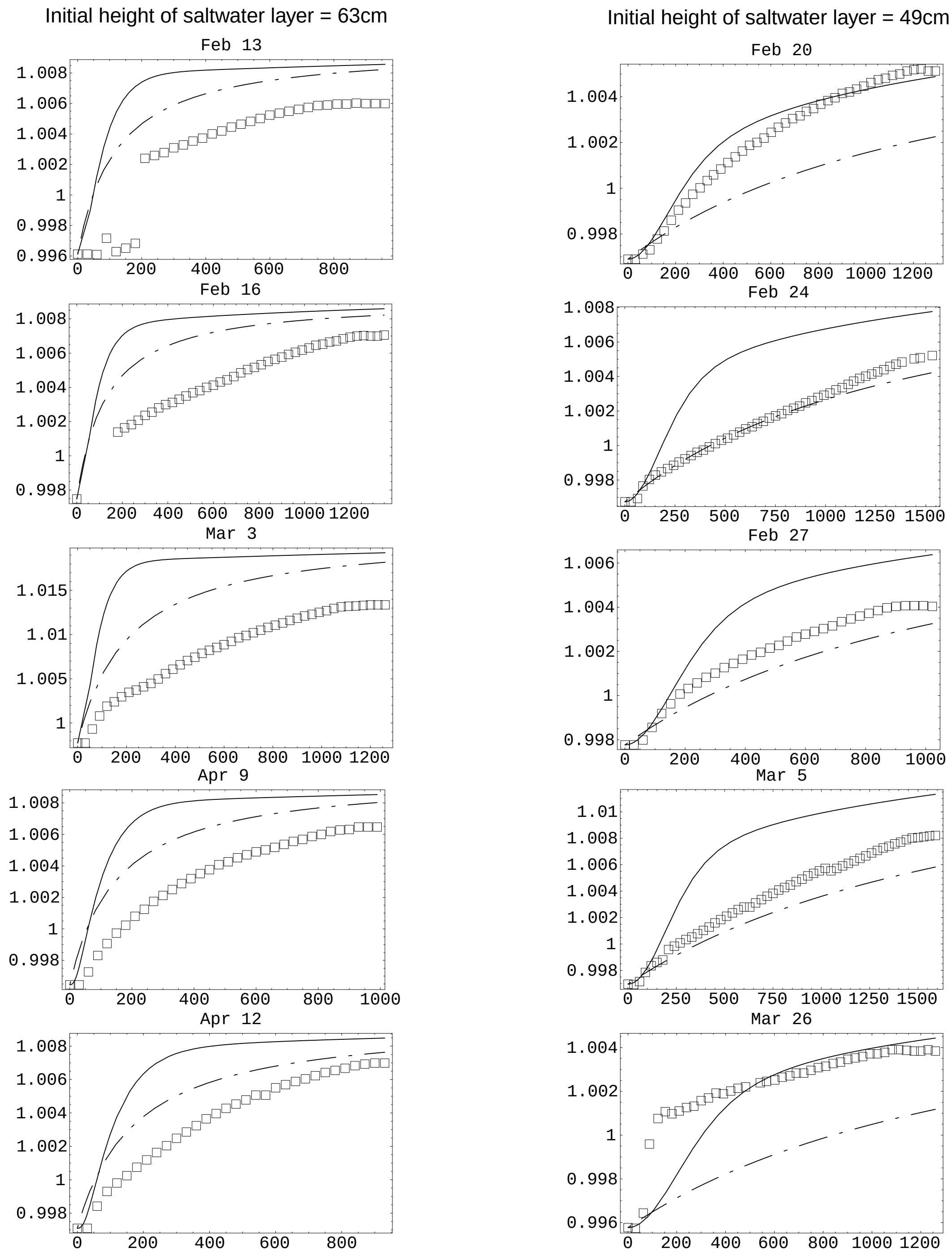

Figure 11: *We compare both the simple model (the entire ODE) (dashed line and* $k = 0.000051$*) and the improved model (solid line and* $k = 0.0040$*) to all the experiments. Note that all the data presented in this thesis tend to level off at the end of the experiment. This leveling off occurs when the pump is turned off but the probe is still collecting data, as the last bit of outflow comes into the bucket.*

7 Discussion

As noted before, we have begun working on finding an appropriate k value, the fraction of kinetic energy transformed into potential energy, for the improved version of the model but have not yet finished the task. We assume k is a constant, but there is a possibility that k is dependent on the height of the lower layer and/or the difference in density between the two layers. This fraction, k, may depend on the difference in densities because as the difference between the two layers decreases, less energy is required to move fluid from the lower layer into the upper layer. The dependence on height is due to the fact that as the lower layer gets further away from the jets, a smaller proportion of the kinetic energy flux is available for entrainment of fluid into the upper layer. When we assume, as we are presently, that k is constant, we are assuming that these two affects cancel each other out. We must also consider that as the density difference between the two layers goes to zero, the height of the lower layer can no longer decrease as the entire tank is homogeneous. The next step, while working with the improved model, is to test what improvements one can get by making k a function of height and/or difference in density.

8 Future Work

First and foremost, we need to continue working with this improved model and work to systematically find a better k value. We will work on understanding the discrepancies between the model and the experiments. We even need to improve the model to incorporate experimental error so we have a better understanding of how much of the error seen is due to the model.

Experimentally, we need to continue collecting data. First, we will repeat the experiments to make sure all the data collected is accurate and repeatable. Then we will work to conduct more experiments using a greater range of parameters and see how our model compares with this data. Also, we need to work on figuring out the best way to measure flux and velocity so we have more accurate values to use in our model. Finally, it would be ideal to try and use vertical jets and to see how this compares with the model and the data collected from horizontal jets.

9 Acknowledgments

We would like to acknowledge and thank Dr. Richard McLaughlin, Dr. David Adalsteinsson, Dr. Gregory Forest, Nicole Abaid, and all the others involved with the UNC-CH Applied Math Fluids Lab for all the time and assistance they have provided us.

References

[1] Benoit Cushman-Roisin. *Introduction to Geophysical Fluid Dynamics*. Prentice Hall. Englewood Cliffs, New Jersey, 1994.

[2] E. J. Strang and H.J.S. Fernando *Entrainment and Mixing in Stratified Shear Flows*. Journal of Fluid Mechanics 428:349-386, 2001.

[3] Nicole Abaid, David Adalsteinsson, Akua Aguapong, and Richard M. McLaughlin. *An Internal Splash: Falling Spheres in Stratified Fluids* UNC Applied Math Preprint

[4] Douglas R. Caldwell and James N. Moum. *Turbulence and mixing in the ocean* US National Report to IUGG, 1991-1994. Rev. Geophys. Volume 33 American Geophysical Union web page. http://earth.agu.org/revgeophys/caldwe01/caldwe01.htm 1995.

[5] Landau et Lifchitz. *Mécanique Des Fluids*. MIR. Moscou, 1971: 275-288.

[6] This fit of salinity versus density was found in the Fluids Lab at the University of North Carolina at Chapel Hill by Jerry Watkins and Dr. Richard McLaughlin.

[7] This fit which allowed us to calculate the amount of salt necessary for a given volume and given salinity was found in the Fluids Lab at the University of North Carolina at Chapel Hill by Ryan McCabe and Dr. Richard McLaughlin.

Appendix: Soret Effect

When preparing an experiment, sometimes as well as a salinity difference in between the two layers, we would also notice a temperature difference. A few times this temperature difference resulted in the salinity increasing slightly before decreasing as height increased (see figure 12). After observing this blip a few times, we realized that this slight increase in salinity only occurred when the temperature of the upper layer was lower then the bottom layer. This effect may possibly be attributed to the Soret Effect discussed in Landau and Lifchitz [5].

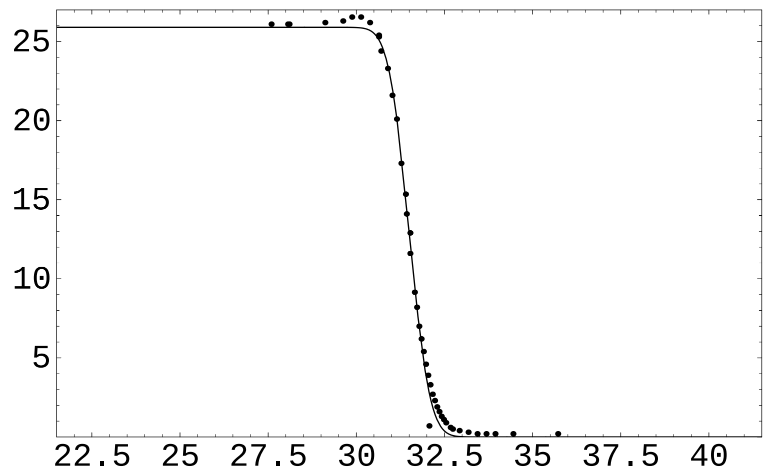

Figure 12: *Plot of height (cm) versus salinity (ppt). Notice the points where the salinity increases slightly before decreasing as height increases. We fit the solution of the diffusion equation to this data.*

This blip disappears over time, if we allow the tank to sit after being stratified, in other words allowing the salt and heat to diffuse from the bottom layer into the upper layer. In one experiment, the tank sat for approximately 20 hours. We fit the solution to the diffusion equation to the profile of the tank initially (see figure 12). Then using this fit as an initial condition for the diffusion equation and putting in the elapsed time, we compare this solution with our data after the elapsed time (see figure 13).

From this graph, we see that the solution is slightly off from the data. The difference that is seen may be due to experimental error. Not having the exact amount of time elapsed and error in placement of the probe might have caused these differences. If we change the time elapsed to 24 hours and shift the interface between the layers by 1 cm, the solution agrees with the data much more accurately (see figure 14).

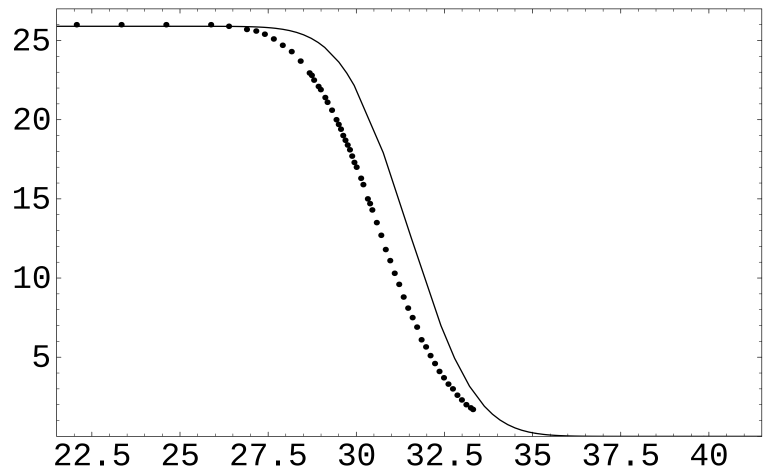

Figure 13: *Plot of height (cm) versus salinity (ppt) after allowing the tank to sit for approximately 20 hours. We compare the solution of the diffusion equation to this profile of the tank.*

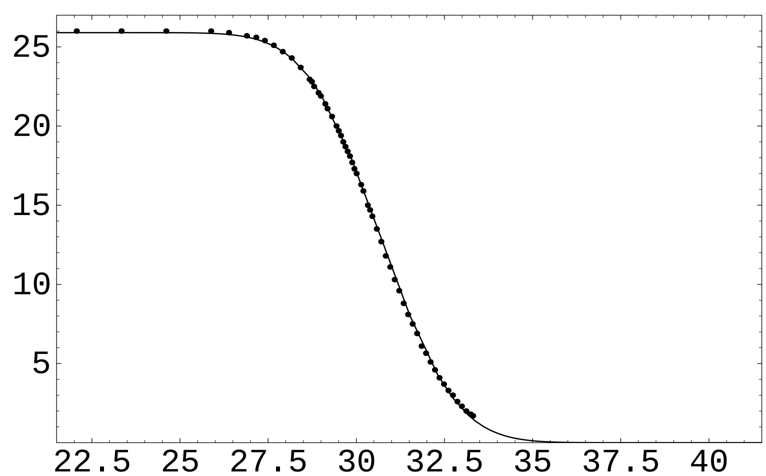

Figure 14: *The solution of the diffusion equation has been adjusted to include an elapsed time of 24 hours and the interface between the layers has been shifted 1 cm.*