On Complexity as Bounded Rationality[1]

- How can we apply complexity in game theory?
- Prisoner's Dilemma
- Bounded Rationality
- General Complexity Result for Games

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Nash Equilibrium

Let G be a game with K players. For each player k we have

- action set A_k of actions available to k
- strategy set $\Delta(A_k)$ of distributions over actions in A_k.
- payoff function $\mu_k : \Pi_{i=1}^{K} A_i \longrightarrow \mathbb{R}$
  - payoffs for mixed strategy $p_k \in \Delta(A_k)$: $\underset{a_k \sim p_k}{E}[\mu_k(\mathbf{a})]$

A **Nash Equilibrium** S is a strategy assignment $(p_1, ..., p_K)$ such that for each player k:

$$\forall q \in \Delta(A_k) : \mu_k(p_k, S_{-k}) \geq \mu_k(q, S_{-k}) \tag{1}$$

n-round Prisoner's Dilemma

	C	D
C	3,3	0,4
D	4,0	1,1

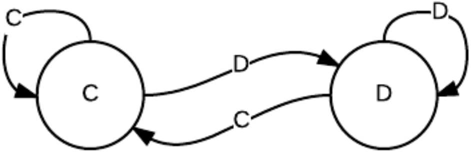
- $A_k^n = [\text{CD}]\{n\}$.
- only Nash equilbrium is (D^n, D^n) via backwards induction

Questions

- Can we avoid the (D^n, D^n) equilibrium by limiting strategic complexity?
- How can we quantify strategic complexity?
- Will players use far more complex strategies for only marginally greater payoffs?

Bounded Rationality

implementation complexity: the number of states required by a finite automaton which implements the strategy



Tit for Tat

Theorem: Let $\epsilon > 0$ and G an n-round Prisoner's Dilemma played by automata. If one of the automata has less than $2^{c_\epsilon n}$ states, then there is a mixed equilibrium with expected payoff for each player at least 3 - ϵ.

Proof Sketch

Lemma: If both players have automata with size at least 2^n then the only equilibrium is (D^n, D^n).

- prove the inverse: if at least one player is limited to sub-exponential automata, then **a mostly collaborative equilibrium is possible**
- construct such an equilibrium:
  - define mixed strategies for each player
  - the automata with size $2^{c_\epsilon n}$ must have its states fully utilized

$\Longrightarrow$ construct a multi-phase mixed strategy for each player

General Complexity Results

- best response:
  - NP
- existence of pure equilibrium:
  - S_2P
- feasible payoffs in mixed equilibrium of zero sum games
  - EXP
- feasible payoffs in mixed equilibrium of general games
  - NEXP