1. Find the limit (if it exists) of the following sequences.

- (1 mark) \( x_n = \frac{2n}{n^2 + 7} \)
- (1 mark) \( x_n = \frac{n^3 + n^2 + n}{2n^3 + n - 2} \)

2. (3 marks) Suppose \( x_0 \in \mathbb{R} \) and \( x_n = \frac{1 + x_{n-1}}{2} \) for every \( n \in \mathbb{N} \). Use the Monotone Convergence Theorem to prove \( x_n \to 1 \).

3. (1 mark) Prove that for any \( x \in \mathbb{R} \), there is a sequence \( q_n \in \mathbb{Q} \) with \( q_n \to x \).
- (2 marks) Prove that there is a sequence \( x_n \) of reals such that
  \[ \{x : \exists \text{ a subsequence } x_{n_k} \text{ with } \lim_{k \to \infty} x_{n_k} = x \} = [0, 1] \]
- (2 marks) Prove that there is no sequence \( x_n \) of reals for which
  \[ \{x : \exists \text{ a subsequence } x_{n_k} \text{ with } \lim_{k \to \infty} x_{n_k} = x \} = (0, 1] \]
  would hold.

4. Decide which of the following statements are true and which are false. Prove the true ones and give counterexample for the false ones.

- (1 mark) If \( \{x_n\} \) is Cauchy and \( \{y_n\} \) is bounded, then \( \{x_n + y_n\} \) is Cauchy.
- (1 mark) If \( \{x_n\} \) and \( \{y_n\} \) are Cauchy with \( y_n \neq 0 \) for all \( n \), then \( \{x_n/y_n\} \) is Cauchy.
- (1 mark) If \( \{x_n\} \) and \( \{y_n\} \) are Cauchy and \( x_n + y_n > 0 \) for all \( n \), then \( \{1/(x_n + y_n)\} \) cannot converge to zero.

The following problems form the extra homework. They will not contribute to your final grade.

5. Recall the definition of the Cantor set from lecture. Use the Nested interval property to prove that the cardinality of the Cantor set is uncountable.