Computational Geometry
Final Exam, May 8, 2002

1. Prove that in the Algebraic Decision Tree model to decide whether \( n \) given numbers are all \textit{distinct} requires at least constant times \( n \log n \) queries (not necessarily comparisons!).

2. Let \( p_1, p_2, \ldots, p_n \) be the vertices of a convex polygon listed in clockwise order. Design a \( O(n) \)-time algorithm for finding the shortest distance between two vertices.

3. Given a set \( P = \{p_1, p_2, \ldots, p_n\} \) of \( n \) points in the plane, for any \( i \), let \( F(p_i) \) denote the set of those point in the plane from which \( p_i \) is at least as far as any other point \( p_j \) (\( j \neq i \)). (The “cells” \( F(p_i), 1 \leq i \leq j \) are usually said to form the so-called \textit{farthest-point Voronoi diagram} of \( P \).) Prove that each cell \( F(p_i) \) is convex.

4. Recall that the \textit{size} of a BSP tree (Binary Space Partition tree) for a set of segments is equal to the total number of segment fragments generated by the splitting lines. An \textit{autopartition} is a BSP-tree in which every splitting line contains one of the segments.

Give an example of a set of \( n \) non-intersecting line segments in the plane, for which a BSP tree of size \( n \) exists, but any autopartition has size at least \( \lceil 4n/3 \rceil \).

5. Let \( S \) be a set of \( n \) circles in the plane.

Outline an algorithm to compute all intersections between the circles in \( O((n + k) \log n) \) time, where \( k \) is the output size, i.e., the number of intersection points.

6. Consider a polygon \( P \) of \( n \) vertices in the plane, which is triangulated by some of its internal diagonals.

Design a \( O(n) \)-time algorithm for coloring the vertices of \( P \) by 3 colors so that no 2 vertices that are connected by an edge or a diagonal get the same color.

Please explain all of your answers! Good luck! - J.P.