CSC 220 Algorithms

Midterm Test A, November 4, 2002

1. Using the method of *QUICKSORT*, find the right ('splitting') position of 35 in the unsorted sequence

$$35, 90, 88, 6, 49, 34, 79, 2, 68.$$

What is the total number of comparisons you made?

2. Suppose $f(1) = 1$ and for all $n \geq 2$,

$$f(n) = 3\left(\sum_{i=1}^{n-1} f(n-i)\right) + 3.$$

What is $f(n)$?

3. Design an algorithm for finding simultaneously the two smallest *and* the two largest elements among 16 distinct numbers using at most 28 comparisons.

4. We have 8 coins that look the same. 6 of them have weight 1, one has weight .99 and one has weight 0.98.

a. Design an algorithm for determining the weights of the coins by a *digital scale*, using at most 4 measurements in the worst case.

b. Is it always possible to identify the 6 good (heavy) coins a two-pan *balance*, using at most 3 measurements?

5. a. Let $f(n), g(n), h(n)$ and $j(n)$ be positive valued functions defined on the set of positive integers, and assume that $f(n) = o(g(n))$ and $h(n) = O(j(n))$. Prove or disprove: $f(n) + h^2(n) = O(g(n) + j^2(n))$.

b. Let $f(n) = \frac{n^2+2n+1}{n+2^{-n}}$ and $g(n) = 100n + 3$ for all $n \geq 0$. Prove or disprove: $f(n) = o(g(n))$.

6. Prove that if there in an algorithm for finding the *second smallest* element in a given set of distinct numbers using at most 100 comparisons, then one can also find in the same time simultaneously the *smallest* and the *second smallest* elements using at most 100 comparisons.

Please explain all of your answers! Good luck! - J.P.