NONCONVENTIONAL LIMIT THEOREMS.

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Abstract. The polynomial ergodic theorem (PET) which attracted substantial attention in ergodic theory studies the limits of expressions having the form $1/N\sum_{n=1}^{N}T^{q_1(n)}f_1\cdots T^{q_\ell(n)}f_\ell$ where T is a weakly mixing measure preserving transformation, f_i's are bounded measurable functions and q_i's are polynomials taking on integer values on the integers. Motivated partially by this result we obtain a central limit theorem for even more general expressions of the form
$1/\sqrt{N}\sum_{n=1}^{N}\big(F\big(X_1(q_1(n)),X_2(q_2(n)),\cdots,X_\ell(q_\ell(n))\big)-\bar{F}\big)$
where X_i's are exponentially fast ψ-mixing bounded stationary processes, F is a Lipschitz continuous function, $\bar{F}=\int Fd(\mu_1\times\cdots\times\mu_\ell)$, μ_j is the distribution of $X_j(0)$, and q_i's are positive functions taking on integer values on integers with some growth conditions which are satisfied, for instance, when q_i's are polynomials of growing degrees. When $F(x_1,...,x_\ell)=x_1x_2\cdots x_\ell$ exponentially fast α-mixing already suffices. This result can be applied in the case when $X_i(n)=T^nf_i$ where T is a mixing subshift of finite type, a hyperbolic diffeomorphism or an expanding transformation taken with a Gibbs invariant measure, as well, as in the case when $X_i(n)=f_i(\xi_n)$ where ξ_n is a Markov chain satisfying the Doeblin condition considered as a stationary process with respect to its invariant measure.