A Large Deviation Result for Pinned Random Walks with Barrier Curves

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Let $X_1, X_2, ...$ be iid with $P(X_i = \pm 1) = 1/2$, with partial sums $S_k = X_1 + ... + X_k$, and let $g : [0, 1] \to \mathcal{R}$ be a concave function with $g(0) = 0$ and $g(1) = \alpha \geq 0$. Let A_n be the event that $S_j/n \geq g(j/n), j = 1, ..., n$, while B_n is the event $\alpha \leq S_n/n \leq \alpha + 2/n$. Then the probability of A_n given B_n decays exponentially, at the asymptotic rate

$$\lim_{n\to\infty} \log P(A_n \mid B_n) = \Psi(\alpha) - \int_0^1 \Psi(g'(x)dx,$$

where Ψ is a function having an explicit formula and is independent of g. This, and more, follows from a non-routine application of Mogulskii's theorem. In particular, the solution to a lattice-path counting problem (that was in fact our initial motivation) can be derived from this result.