Linear Algebra Final

August 6, 2010

1. Find the projection of $\begin{bmatrix} 4 \\ 3 \\ 1 \\ 0 \end{bmatrix}$ onto the columnspace of $\begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}$

2. Let $A = \begin{bmatrix} 2 & 1 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$.

   How many solutions are there to $A\mathbf{x} = \mathbf{b}$?

3. Let $\mathbf{u}_1 = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$, and $\mathbf{u}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Let $H = span(\mathbf{u}_1, \mathbf{u}_2)$.

   - What is the dimension of H?
   - Let $T(\mathbf{x}) = proj_H(\mathbf{x})$. Find the matrix A so that $T(\mathbf{x}) = A\mathbf{x}$.
   - Just by thinking (not multiplication or diagonalizing) compute A^{500}.
   - Find the eigenvalues of A.
   - Find a basis for the eigenspace of the *largest* eigenvalue.
   - Can you conclude something in general about the eigenvalues and eigenspaces of orthogonal projection transformations (like T)?

4. Let A be orthogonally diagonalizable - i.e., there is an *orthogonal* matrix P and a diagonal matrix D so that $A = PDP^{-1}$. Prove that A is a symmetric matrix.

5. Let V be the abstract vector space of all 10×10 matrices with the usual scalar multiplication and additon of two matrices. Which of the following is a *subspace* of V? Give a reason for each one that is *not* a subspace. (All matrices below are 10×10).

   - The set of all invertible matrices

- The set of all matrices with upper right hand entry 0
- The set of all matrices with first column equal the sum of the second and third columns
- The set of all matrices with positive determinant
- The set of all matrices with integer entries
- The set of all matrices whose $(1,1)$ entry is $\geq$ to its $(1,2)$ entry

Extra Credit: What is the dimension of V?

Extra extra credit: What is a basis for V?

6. Suppose A and B are similar matrices. (i.e. $A = PBP^{-1}$). Briefly explain your answers (at most 2 sentences).
   - If A is onto as a linear transformation, is B onto?
   - If A is one-to-one, is B one-to-one?
   - If $\lambda = 5$ is an eigenvalue of A, is $\lambda = 5$ an eigenvalue of B?
   - If $\mathbf{x}$ is an eigenvector of A, is $\mathbf{x}$ an eigenvector for B?
   - If $\mathbf{x}$ solves $A\mathbf{x} = \mathbf{b}$, does $\mathbf{x}$ solve $B\mathbf{x} = \mathbf{b}$? (Same $\mathbf{x}$, same $\mathbf{b}$)
7. Let $\mathbf{v}$ be a vector in R^n, $\mathbf{v} \neq \mathbf{0}$.
   - What type of object is $\mathbf{v}\mathbf{v}^T$? (scalar, vector, matrix, subspace)
   - Can you compute the nullity (dimension of nullspace) of $\mathbf{v}\mathbf{v}^T$?
   - If so, what is it? If not, why not?