1. **Improving the Sudan list-decoding algorithm**: In class we saw how to list decode RS codes from agreements as small as $2\sqrt{kn}$. Improve this to $\sqrt{2kn}$. The best-known algorithm, due to Guruswami and Sudan, needs only $\sqrt{kn}$ agreement and is based on multiplicities. See Sudan’s lecture notes or Guruswami’s thesis.

2. **Yekhanin’s approach to locally decodable codes**: A binary code $C : \{0, 1\}^n \rightarrow \{0, 1\}^N$ is said to be $(q, \delta, \varepsilon)$-locally decodable if there exists a randomized decoding algorithm $A$ that on input $y \in \{0, 1\}^N$ and $i \in [n]$ where $y$ satisfies $\delta(y, C(x)) \leq \delta$ for some $x \in \{0, 1\}^n$ outputs $x_i$ with probability at least $1 - \varepsilon$ and makes at most $q$ queries to $y$. In class we saw a simple $(2, \frac{1}{4} - \varepsilon, \frac{1}{2} - 2\varepsilon)$-local decoder for Hadamard codes.

**Definition 1** Let $N, R, n \geq 1$ be some integers. For each $i \in [n], r \in [R]$, let $T_i$ and $Q_{ir}$ be subsets of $[N]$. We say that $T_i$ and $Q_{ir}$ form a $(q, n, N, R, s)$ regular intersecting family if the following holds:

(a) $q$ is odd;
(b) For all $i \in [n], |T_i| = s$;
(c) For all $i \in [n]$ and $r \in [R], Q_{ir} \subseteq T_i$ and $|Q_{ir}| = q$;
(d) For all $i \in [n]$ and $w \in T_i, \{r \in [R] \mid w \in Q_{ir}\} = Rq/s$ (i.e., $T_i$ is uniformly covered by the $Q_{ir}$);
(e) For all $i, j \in [n], i \neq j$ and $r \in [R], |Q_{ir} \cap T_j| \equiv 0 \mod 2$.

Show how to use a $(q, n, N, R, s)$ regular intersecting family to construct binary linear code encoding $n$ bits into $N$ bits that is $(q, \delta, \delta Nq/s)$ locally decodable for all $\delta > 0$.

3. **One small step in Yekhanin’s construction of a regular intersecting family**: Let $p$ be an odd prime number and $m \geq p - 1$ be an integer. Define $n = \binom{m}{p-1}$. Show that there exist two families of vectors $\{u_1, \ldots, u_n\}$ and $\{v_1, \ldots, v_n\}$ in $\mathbb{F}_p^m$ such that

- for all $i \in [n], \langle u_i, v_i \rangle = 0$;
- for all $i, j \in [n], i \neq j$, we have $\langle u_i, v_j \rangle \neq 0$. 
