In this problem, you will use a PRG to implement what we’ll call a secure “locking” scheme. A locking scheme is a protocol between two players, a locker $L$ and a verifier $V$. It allows $L$ to lock itself into one of two choices ($0$ or $1$) without $V$ knowing which choice was made, then later reveal its choice. The protocol works in two phases: in the first “locking” phase, $L$ and $V$ exchange some messages, which result in $L$ being bound to its (secret) choice bit. In the second “unlocking” phase, $L$ reveals its choice bit and some additional information, which allows $V$ to check consistency with the earlier messages.

We define the following model for a locking scheme, in which the locking phase consists of an initial message from the verifier, followed by a response from the locker.

- The verifier $V()$ is a PPT algorithm that takes no input (except for the implicit security parameter $1^n$ and its random coins) and outputs some message $v \in \{0, 1\}^*$.  
- The locker $L(\sigma, v; r_L)$ is a PPT algorithm that takes a choice bit $\sigma \in \{0, 1\}$, the verifier’s initial message $v$, and random coins $r_L$, and outputs some message $\ell \in \{0, 1\}^*$. 

In the unlocking phase, the locker simply reveals $\sigma$ and $r_L$, and the verifier checks that $\ell = L(\sigma, v; r_L)$.

(a) (3 points) A secure locking scheme should be “hiding,” i.e., a malicious (but computationally bounded) verifier $V^*$ should not be able to learn anything about the honest locker $L$’s choice bit $\sigma$, no matter what initial message $v^*$ the malicious verifier sent.

Using the notion of indistinguishability, give a formal definition of this hiding property.

(b) (3 points) A secure locking scheme should also be “binding” against even a computationally unbounded malicious locker $L^*$. That is, there should not exist any $\ell^*$ that can successfully be unlocked as both choice bits $\sigma \in \{0, 1\}$, except with negligible probability over the choice of the honest verifier $V$’s initial message $v$.

Give a formal definition of this binding property.

(c) (3 points) Let $G$ be any length-tripling function, i.e., one for which $|G(x)| = 3|x|$ for every $x \in \{0, 1\}^*$. Give an upper bound on the probability, over the choice of a random $3n$-bit string $R$, that there exist two inputs $x_1, x_2 \in \{0, 1\}^n$ such that $G(x_1) \oplus G(x_2) = R$.

(d) (6 points) Let $G$ be a length-tripling PRG (which we have seen can be obtained from any PRG). Use $G$ to construct a secure locking scheme, and prove that it is both hiding and binding according to your definitions. [I need a hint! (ID 19922)]

(e) (0 points) Think how using the locking scheme two remote parties can toss a fair coin over the Internet, even if one of them is dishonest. For more discussion and cool applications, see Dodis’s lecture 14.

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1From Peikert