1. Let $I, J \subset \mathbb{C}[x_0, \ldots, x_n]$ be irrelevant ideals. Show that $I + J$ and $J \cap J$ are also irrelevant.

2. Consider the homogeneous ideal $J := \langle x_0^2x_1, x_1^3, x_1x_2 \rangle$. For $i = 0, 1, 2$ let $I_i$ be the dehomogenization of $J$ with respect to $x_i$ and $J_i$ the homogenization of $I_i$. Compute each $I_i$ and $J_i$. Show that $J \subseteq J_0 \cap J_1 \cap J_2$. Show that $(J : \langle x_0, x_1, x_2 \rangle) \neq J$.

3. Find the points of intersection and multiplicities at the intersections of the plane curves given by:

\[ y^2z - x(x - 2z)(x + z) = y^2 + x^2 - 2xz = 0. \]

4. Compute the Hilbert polynomial of

\[ J := \langle x_0x_1, x_2^2 \rangle \subset \mathbb{C}[x_0, x_1, x_2]. \]

5. Compute the Hilbert polynomial of

- an irreducible conic in $\mathbb{P}^3$ and
- a union of two nonintersecting lines in $\mathbb{P}^3$. 