Functions

(1986) Let

\[ f(x, y) = \begin{cases} \frac{x^2y}{x^2+y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases} \]

If \( f \) continuous at \((0,0)\)? Do \( f_x \) and \( f_y \) exist? If \( f \) differentiable?

(1988) Is the function \( f(x) = \sqrt{x}\sin\left(\frac{1}{x}\right), \; x > 0 \), uniformly continuous on the interval \((0,1])\)? Prove your answer.

(1992) Assume that \( f(x) \) has second order derivatives on \([a,b]\), and \( f'(a) = f'(b) = 0 \). Prove that there exists a point \( c \in [a,b] \) such that

\[ f''(c) \geq \frac{4}{(a-b)^2}|f(b) - f(a)|. \]