Abstract
Fast Solvers and Schwarz Preconditioners for Spectral Nédélec Element
Discretizations of a Model Problem in $H(\text{curl})$
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Recently, Toselli and others proposed Schwarz preconditioners for a model problem
\[ \eta_1 (\vec{u}, \vec{v})_{L^2(\Omega)} + \eta_2 (\text{curl} \; \vec{u}, \text{curl} \; \vec{v})_{(L^2(\Omega))^d} = f(v) \]
in $H(\text{curl})$, discretized with $h$-version Nédélec elements. We will present both theoretical and numerical results for similar methods applied to spectral Nédélec elements.

Spectral element discretizations have been used very successfully for problems in computational fluid dynamics, and hold great promise for the solution of problems in computational electro-magnetics as they combine superior approximation properties, geometric flexibility and a special structure that allows for fast, tensorized, algorithms.

Problems in $H(\text{curl})$ require $H(\text{curl})$-conforming elements in which only the tangential components across the elements have to match. The use of standard, $H^1$-conforming, elements introduces spurious eigenvalues. Therefore, Nédélec type edge elements have to be used.

We use the $hN$-extension of Nédélec edge elements, and we introduce new spectral element type degrees of freedom allowing fast, tensorized, algorithms. We generalize the elements to arbitrary degrees with possibly different degrees in different directions, and provide for numerical integration of arbitrary order. These elements are then used to discretize the model problem in two and three dimensions.

We developed fast direct solvers for the model problem in rectangular domains, and we use them as local solvers in Schwarz methods.

We extend the theory in Toselli (Numer. Math., 86(4):733–752, 2000) for overlapping Schwarz methods to the spectral Nédélec element case. We reduce the proof of the condition number estimate to three basic estimates, and present theoretical and numerical results on those estimates. The technique of the proof works in both the two-dimensional and three-dimensional case.

We also present numerical results for one-level and two-level methods in two dimensions.