Show $\gamma$ is well defined.

Suppose $C_1$ starts at $0$ and travels up to $(x, y_0)$. 

Suppose $C_2$ is another curve that starts at $0$ and travels up to $(x_0, y_0)$. 

For $\gamma$ to be well defined we must show
\[ \oint_{C_2} F(r) \cdot dr = \oint_{C_1} F(r) \cdot dr \]

But the curve $C$ defined by traveling along $C_1$ to $(x_0, y_0)$ and then backwards along $C_2$ to $0$ is a closed curve.

Thus $C = C_1 - C_2$ is closed and since
\[ \oint_{C} F(r) \cdot dr = 0 \Rightarrow \oint_{C_1 - C_2} F(r) \cdot dr = 0 \]

Thus $\gamma$ is well defined.