1. An archer is known to enjoy a drink before each competition (to calm nerves); unfortunately, it makes him less accurate.

Let $X$ denote the amount he drinks, and suppose $X$ is a continuous random variable, uniformly distributed over $[0, 3]$. Let $Y$ denote his accuracy, as measured by how close he can place the arrow to the center of the target. $Y$ is also a continuous random variable; we assume that, given he had $X = x$ drinks, $Y$ is uniformly distributed over $[0, x]$.

(a) Find the joint density $f_{X,Y}$.

(b) Find the marginal density $f_Y$.

(c) Given that $Y = 2$, what is the conditional probability density of $X$?

(d) What is the most likely amount he drank, i.e. what value of $x$ maximizes $f_{X|Y}(x|2)$?

(Notice that in part 1c, we are essentially using the continuous version of Bayes’s formula.)