Financial Modeling of the Equity Market
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FJF
To my wife Donna and my children, Francesco, Patricia, and Karly

SMF
To the memory of Bertrand Russell to whom I owe the foundation of my intellectual development

PNK
To my best friend, my wife, and my love—Carmen
## Contents

Preface xiii  
Acknowledgments xvii  
About the Authors xix  

**CHAPTER 1**  
Introduction 1  
Historical Perspective on the Financial Modeling of the Equity Market 1  
Central Themes of the Book 8  
Organization of the Book 9  

**PART ONE**  
Portfolio Allocation: Classical Theory and Modern Extensions 13  

**CHAPTER 2**  
Mean-Variance Analysis and Modern Portfolio Theory 15  
The Benefits of Diversification 17  
Mean-Variance Analysis: Overview 19  
Classical Framework for Mean-Variance Optimization 22  
The Capital Market Line 34  
Selection of the Optimal Portfolio When there is a Risk-Free Asset 40  
More on Utility Functions: A General Framework for Portfolio Choice 42  
Summary 48  

**CHAPTER 3**  
Transaction and Trading Costs 51  
A Taxonomy of Transaction Costs 52  
Liquidity and Transaction Costs 60  
Market Impact Measurements and Empirical Findings 63  
Forecasting and Modeling Market Impact 68  
Incorporating Transaction Costs in Asset-Allocation Models 74  
Optimal Trading 80
Contents

Integrated Portfolio Management:
Beyond Expected Return and Portfolio Risk 82
Summary 85

CHAPTER 4
Applying the Portfolio Selection Framework in Practice 87
Rebalancing in the Mean-Variance Optimization Framework 88
Portfolio Constraints Commonly Used in Practice 100
Summary 113

CHAPTER 5
Incorporating Higher Moments and Extreme Risk Measures 115
Dispersion and Downside Measures 116
Portfolio Selection with Higher Moments through Expansions of Utility 131
Polynomial Goal Programming for Portfolio
Optimization with Higher Moments 139
Some Remarks on the Estimation of Higher Moments 141
The Approach of Malevergne and Sornette 142
Summary 147

CHAPTER 6
Mathematical and Numerical Optimization 149
Mathematical Programming 150
Necessary Conditions for Optimality for
Continuous Optimization Problems 158
How Do Optimization Algorithms Work? 159
Optimization Software 176
Practical Considerations when Using Optimization Software 180
Summary 187

PART TWO
Managing Uncertainty in Practice 189

CHAPTER 7
Equity Price Models 191
Definitions 191
Theoretical and Econometric Models 193
Random Walk Models 194
General Equilibrium Theories 207
Capital Asset Pricing Model (CAPM) 208
Arbitrage Pricing Theory (APT) 212
Summary 213
CHAPTER 8
Forecasting Expected Return and Risk 215
- Dividend Discount and Residual Income Valuation Models 217
- The Sample Mean and Covariance Estimator 222
- Random Matrices 231
- Arbitrage Pricing Theory and Factor Models 234
- Factor Models in Practice 241
- Factor Models in Practice: An Example 245
- Other Approaches to Volatility Estimation 259
- Application to Investment Strategies and Proprietary Trading 264
- Summary 265

CHAPTER 9
Robust Frameworks for Estimation and Portfolio Allocation 267
- Robust Frameworks for Estimation and Portfolio Allocation 267
- Practical Problems Encountered in Mean-Variance Optimization 269
- Shrinkage Estimation 275
- Bayesian Approaches 281
- Incorporating Estimation Error and Uncertainty in the Portfolio Allocation Process 304
- Summary 318

PART THREE
Dynamic Models for Equity Prices 321

CHAPTER 10
Feedback and Predictors in Stock Markets 323
- Random Walk Models and Their Shortcomings 323
- Time Diversification 333
- A Multiagent Economy: Effects of Agent Heterogeneity and Interactions 339
- Market Predictors 343
- Time Aggregation 345
- Summary 345

CHAPTER 11
Individual Price Processes: Univariate Models 347
- Time Series Concepts 348
- Digression on White Noise and Martingale Difference Sequences 350
- The Lag Operator L 353
- Univariate Autoregressive Moving Average (ARMA) Models 353
- Stationarity Conditions 354
- Auto Correlations at Different Lags 357
- Solutions of an AR(p) Process 358
MA(q) Moving Average Models 362
ARMA(p,q) Models 363
Integrated Processes 364
Summary 365

CHAPTER 12
Multivariate Models 367
Dynamic Models: A Historical Perspective 368
Vector Autoregressive Models 370
Vector Autoregressive Moving Average Models (VARMA) 385
Distributional Properties 386
Cointegration 386
Stochastic and Deterministic Cointegration 392
Common Trends 393
Error Correction Models 395
Forecasting with VaR Models 396
State-Space Models 397
Autoregressive Distributed Lag Models 399
Dynamic Factor Models 402
The ARCH/GARCH Family of Models 402
Nonlinear Markov-Switching Models 404
Summary 405

CHAPTER 13
Model Selection and its Pitfalls 407
Model Selection and Estimation 407
The (Machine) Learning Approach to Model Selection 410
Sample Size and Model Complexity 415
Dangerous Patterns of Behavior 419
Data Snooping 424
Survivorship Biases and Other Sample Defects 426
Moving Training Windows 428
Model Risk 430
Model Selection in a Nutshell 431
Summary 433

PART FOUR
Model Estimation and Model Risk Mitigation 437

CHAPTER 14
Estimation of Regression Models 439
Probability Theory and Statistics 439
Populations of Prices and Returns 442
Contents

Estimation at Work 444
Estimators 445
Sampling Distributions 446
Critical Values and Confidence Intervals 450
Maximum Likelihood, OLS, and Regressions 450
The Fisher Information Matrix and the Cramer-Rao Bound 453
Regressions 454
Linear Regressions 456
Sampling Distributions of Regressions 464
Relaxing the Normality and Uncorrelated Noise Assumptions 468
Pitfalls of Regressions 469
The Method of Moments and its Generalizations 471
Summary 475

CHAPTER 15

**Estimation of Linear Dynamic Models** 477
An Approach to Estimation 477
Unit Root Testing 478
Estimation of Linear Regression Models 479
Estimation of Stable Vector Autoregressive (VAR) Models 482
Estimating the Number of Lags 499
Autocorrelation and Distributional Properties of Residuals 501
Stationary Autoregressive Distributed Lag Models 502
Applying Stable VAR Processes to Financial Econometrics 503
Stationary Dynamic Factor Models 506
Estimation of Nonstationary VAR Models 509
Estimation with Canonical Correlations 520
Estimation with Principal Component Analysis 521
Estimation with the Eigenvalues of the Companion Matrix 523
Estimation with Subspace Methods and Dynamic Factor Analysis 524
Application of Cointegration Methods to the Analysis of Predictors 524
Summary 525

CHAPTER 16

**Estimation of Hidden Variable Models** 529
Estimation of State-Space Models 530
Estimation of Factor Analytic Models 543
Estimation Methods for Markov-Switching Models 546
Applications 548
Summary 552

CHAPTER 17

**Model Risk and its Mitigation** 555
Sources of Model Risk 555
The Information Theory Approach to Model Risk 558
Bayesian Modeling 563
Model Averaging and the Shrinkage Approach to Model Risk 573
Random Coefficients Models 574
Summary 575

APPENDICES 577

APPENDIX A
Difference Equations 579
Homogeneous Difference Equations 579
Nonhomogeneous Difference Equations 588
Systems of Linear Difference Equations 594
Systems of Homogeneous Linear Difference Equations 595

APPENDIX B
Correlations, Regressions, and Copulas 603
Probability Density Function, Marginal Density, and Conditional Density 603
Expectations and Conditional Expectations 604
Variances, Covariances, and Correlations 606
Normal Distributions 608
Regression 610
Multivariate Extension 612
Multiple and Multivariate Regressions 613
Canonical Correlations 615
Copula Functions 616

APPENDIX C
Data Description 619

INDEX 629
This book is about financial modeling for equity asset management. We take a broad view of financial modeling, encompassing pure modeling as well as model engineering and financial optimization. Our perspective is that of an asset management firm. When reasoning and making decisions about modeling, a firm needs to grasp all the aspects related to modeling. This includes not only the mathematical models per se but also methods for model estimation, the optimization process that translates model forecasts into active strategies, and methods that help mitigate eventual inadequacies of the models being used.

Our perspective is similar to that of physical engineering, where the knowledge of a few abstract laws of physics is a far cry from building an automobile or an airplane. We broadly define financial modeling as theoretical financial and mathematical principles as well as statistical methods that allow for representing and forecasting financial data, procedures for estimating and testing these representations, and methods for engineering and optimizing financial strategies. Without a methodology for engineering, estimating, and testing financial strategies, a financial model is of little use.

In this book we offer an up-to-date treatment of financial modeling for asset management, presenting and discussing a number of developments at the forefront of equity modeling technology: robust estimation, robust optimization, the analysis of transaction costs, linear and non-linear dynamic models, and model risk mitigation techniques.

Since the downturn in the U.S. equity market in 2002, there has been an increased use of financial modeling and optimization in equity portfolio management. Under pressure to boost returns and reduce costs, asset management firms have begun to look with increasing attention at quantitative techniques. Not only has the diffusion of quantitative methods in equity portfolio management broadened since the turn of the century, but the variety of models and depth of use have also increased.

Three trends are worth pointing out. First, there is a greater use of predictive models. Predictive models assume that it is possible to make conditional forecasts of expected returns, an objective that was previously considered not achievable by classical financial theory. Second,
order to exploit forecasts, optimization techniques are now being used. Previously, optimization technologies were considered too brittle for safe deployment in asset management. Third, as a consequence of a greater use of predictive models and optimization, there is a growing interest in “robust” methods—particularly methods for robust estimation and robust optimization—as well as a heightened attention to the analysis of transaction costs.

Two technology trends have also facilitated the deployment of modeling in equity asset management. First, the continuous decrease in the cost of computers coupled with a parallel increase in computational power makes the necessary computing power affordable even to small firms. Second, statistical software packages now offer a broad variety of general and specialized econometric building blocks. The availability of these software packages proved to be a powerful enabler for the deployment of modeling.

The book is divided into four parts. In Part One we cover modern portfolio theory, numerical optimization methods, the analysis of transaction costs, and the handling of nonnormal distributions in portfolio allocation applications through the consideration of higher moments. We present important recent theoretical advances as well as the basic modeling techniques. In Part One these methods are applied in the classical one-period mean-variance and utility-maximization frameworks. This allows us to give an up-to-date treatment of modern portfolio theory and to explain new methods of analysis of transaction costs, numerical optimization, and the handling of higher moments in a unified and concrete framework.

In Part Two we introduce robust methodologies. As mentioned above, robust techniques have become fundamental in the practical deployment of modern portfolio theory. We discuss both the classical and more recent methods for forecasting expected return and risk. In particular, we address topics including dimensionality reduction and the robust estimation of the covariance matrix of returns. Part Two provides a comprehensive presentation of robust methodologies for estimation and optimization.

In Part Three we discuss the motivation for adopting predictive models and present several families of models. We begin with an analysis of the empirical evidence of feedbacks in financial markets. We then describe the statistical properties of models that allow to capture these feedbacks, including regressive and autoregressive models, state-space models, and nonlinear hidden variable, regime-switching models. We discuss cointegration and its many different representations, including dynamic factor analysis. We also elaborate on the process and the pitfalls of the model selection process.
In Part Four we discuss current methods for estimating dynamic models. We close with a discussion on how to mitigate model risk in a dynamic framework.

Two appendices provide complementary mathematical details for the interested reader. Appendix A discusses solutions of difference equations. Appendix B presents a number of mathematical facts on regressions, correlations, and copulas. In several chapters throughout the book we make use of the MSCI World Index and its individual constituents (country indices) in various illustrations. Appendix C provides some basic statistics and properties of this data set.

The purpose of this book is to serve as a working tool for practitioners who use financial modeling in their work and for students who are pursuing careers in finance. Since most of the subjects are advanced in nature, we have tried to offer an intuitive and simplified treatment of most mathematical topics, although at no time have we compromised mathematical rigor. When we feel the subject is too technical, we offer references to the original work. In summary, we feel the book should be of interest to practitioners, students, and researchers who need an updated and integrated view of equity modeling.

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