Calculus I - Midterm

Write your answers on separate sheets of paper. Show all of your work. No calculators!

1) (10 points) Evaluate the limits and justify your answers. If the limit does not exist, explain why.

- \( \lim_{x \to \pi/2} \tan(\sqrt{x}) \)
- \( \lim_{x \to 5} \frac{x^2 - 5x}{x^2 - 3x - 10} \)
- \( \lim_{x \to 3} \frac{x - 3}{x^2 - 6x + 9} \)
- \( \lim_{x \to \infty} x(\sqrt{x^2 + 6} - x) \)
- \( \lim_{x \to 4} \frac{x^2 - 16}{|x - 4|} \)

2) (6 points)

- Given a function, \( f(x) \), write the limit definition of the derivative, \( f'(x) \).

- Use your definition to compute the derivative of \( f(x) = x^2 + 7x + 2 \).

- Write an equation for the tangent line to \( f(x) = x^2 + 7x + 2 \) through the point \((a, f(a))\). In other words, give the linearization of \( f \) at \( a \).

3) (6 points) Let \( h(x) = (7 - x^2)^{1/3} \).

- What is the domain of \( h \)?

- Where is \( h \) continuous?

- Where is \( h \) differentiable?
4)(15 points) Find the derivative.

- \( f(x) = 4(x^3 + 11) \)

- \( f(x) = \sqrt{x}(x^2 + 4x) \)

- \( f(x) = \frac{x^2 + 6x + 1}{x} \)

- \( f(x) = (1 + x \sin(x))^2 \)

- \( f(x) = \frac{\cos(x^2)}{2x^4} \)

5)(5 points) Consider the curve given by:

\[
4x^2 + xy + 3y^2 = 8
\]

Use implicit differentiation to find an equation for the tangent line at the point (1, 1).

6)(8 points) Let \( g(x) \) be defined by:

\[
g(x) = \begin{cases} 
  x^2 \sin(1/x) & \text{if } x \neq 0 \\
  0 & \text{if } x = 0
\end{cases}
\]

- Is \( g \) continuous at \( x = 0 \)?

- Is \( g \) differentiable at \( x = 0 \)? (Use the definition of the derivative as a limit)

- Compute \( g'(x) \) for \( x \neq 0 \). (Use differentiation rules)

- Is \( g'(x) \) continuous at 0?