1. Let
\[ \mu_{B,k}(a + p^N Z_p) := p^{N(k-1)} B_k(\frac{a}{p^N}), \quad k \in \mathbb{N}. \]
Show that \( \mu_{B,k} \) are distributions on \( \mathbb{Z}_p \).

2. Compute \( \mu_{B,k}(Z_p) \), \( \mu_{B,k}(pZ_p) \), \( \mu_{B,k}(\mathbb{Z}_p^*) \).

3. Define \( \Lambda(n) = \log(p) \) if \( n = p^a \), and \( \Lambda(n) = 0 \), otherwise. Show that for \( \Re(s) > 1 \) one has
\[ \frac{\zeta'(s)}{\zeta(s)} = - \sum_{n \geq 1} \frac{\Lambda(n)}{n^s}. \]

4. Let \( A_n \) (resp. \( G_n \)) be the arithmetic (resp. geometric) mean of
\[ \left( \binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n} \right). \]
Show that
\[ \lim_{n \to \infty} \sqrt[n]{A_n} = 2 \quad \text{and} \quad \lim_{n \to \infty} \sqrt[n]{G_n} = \sqrt{e}. \]

5. Let
\[ f(s) := \sum_{n=1}^{\infty} \frac{a_n}{n^s} \]
be absolutely convergent for \( \Re(s) > \sigma > 0 \). Put
\[ g(x) := \sum_{n=1}^{\infty} a_n e^{-nx}. \]
Show that, for \( \Re(s) > \sigma \), one has
\[ f(s) \Gamma(s) = \int_0^{\infty} g(x) x^{s-1} dx. \]